QUANTITATIVE TECHNIQUES FOR BUSINESS

(BBA4 C04)

STUDY MATERIAL

IV SEMESTER

BBA

(2019 Admission)

UNIVERSITY OF CALICUT
SCHOOL OF DISTANCE EDUCATION
CALICUT UNIVERSITY P.O
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UNIVERSITY OF CALICUT

SCHOOL OF DISTANCE EDUCATION

BBA4C05 Quantitative Techniques for Business

Complementary Course – BBA 2019 Admission

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Syllabus

Complementary Course

BACHELOR OF BUSINESS ADMINISTRATION

BBA4C04 - QUANTITATIVE TECHNIQUES FOR BUSINESS

Time: 5 Hours per week Credits: 4

Internal 20: External 80

Objective: To familiarise student with the use quantitative techniques in managerial decision making.

Learning Outcome : On completing the course students will be able to

Understand and develop insights and knowledge base of various concepts of Quantitative Techniques.

Develop skills for effectively analyze and apply Quantitative Techniques in decision making.

Module I : Quantitative Techniques: Introduction - Meaning and Definition – Classification of QT -QT and other disciplines – Application of QT in business – Limitations. **05 Hours**

Module II : Time Series and Index Number: Meaning and Significance – Utility, Components of Time Series- Measurement of Trend: Method of Least Squares, Parabolic Trend and Logarithmic Trend- Index Numbers:Meaning and Significance, Problems in Construction of Index Numbers, Methods of Constructing Index Numbers – Weighted and Unweighted, Test of Adequacy of Index Numbers, Chain Index Numbers.

20Hours

Module III : Correlation and Regression Analysis: Correlation:- Meaning, significance and types; Methods of Simple correlation - Karl Pearson"s coefficient of correlation, Spearman"s Rank correlation - Regression -Meaning and significance; Regression vs. Correlation - Linear Regression, Regression lines (X on Y, Y on X) and Standard error of estimate.

20 Hours

Module IV : Probability: –Concept of Probability—Meaning and Definition—Approaches to Probability Theorems of Probability—Addition Theorem—

Multiplication Theorem—Conditional Probability—Inverse Probability—Bayes' Theorem - Sets Theory:Meaning of Set - Set Operation – Venn Diagrams.

20 Hours

Module V : Theoretical Distribution: Binomial Distribution – Basic Assumptions and Characteristics – Fitting of Binomial Distribution – Poisson Distribution – Characteristics - Fitting of Poisson Distribution – Normal Distribution – Features and Properties – Standard Normal Curve.

15 Hours

(Theory and problems may be in the ratio of 30% and 70% respectively)

Reference Books:

Richard I. Levin and David S. Rubin, Statistics for Management, Prentice Hall ofIndia, latest edition.

S.P.Gupta, Statistical Methods, Sultan Chand.

Sanchetti and Kapoor, Statistics, Sultan Chand.

G.C.Beri, Statistics For Managemet,Tata McGraw Hill.

J.K. Sharma, Business Statstics:Pearson.

Anderson Sweeney Williams, Statistics for Business and Economics, Thomson.

Levine Krebiel&Bevenson, Business Statistics, Pearson edition, Delhi.

Module I : Quantitative Techniques

Quantitative techniques may be defined as those techniques which provide the decision makes a systematic and powerful means of analysis, based on quantitative data. It is a scientific method employed for problem solving and decision making by the management. With the help of quantitative techniques, the decision maker is able to explore policies for attaining the predetermined objectives. In short, quantitative techniques are inevitable in decision-making process.

Classification of Quantitative Techniques:

There are different types of quantitative techniques. We can classify them into three categories. They are:

Mathematical Quantitative Techniques

Statistical Quantitative Techniques

Programming Quantitative Techniques

Mathematical Quantitative Techcniques:

A technique in which quantitative data are used along with the principles of mathematics is known as mathematical quantitative techniques. Mathematical quantitative techniques involve:

1.Permutations and Combinations:

Permutation means arrangement of objects in a definite order. The number of arrangements depends upon the total number of objects and the number of objects taken at a time for arrangement. The number of permutations or arrangements is calculated by using the following formula:-

$$nP_r = \frac{n!}{(n-r)!}$$

Combination means selection or grouping objects without considering their order. The number of combinations is calculated by using the following formula:-

$$nC_r = \frac{n!}{(n-r)!}$$

2. Set Theory:-

Set theory is a modern mathematical device which solves various types of critical problems.

3. Matrix Algebra:

Matrix is an orderly arrangement of certain given numbers or symbols in rows and columns. It is a mathematical device of finding out the results of different types of algebraic operations on the basis of the relevant matrices.

4. Determinants:

It is a powerful device developed over the matrix algebra. This device is used for finding out values of different variables connected with a number of simultaneous equations.

5. Differentiation:

It is a mathematical process of finding out changes in the dependent variable with reference to a small change in the independent variable.

6. Integration:

Integration is the reverse process of differentiation.

7. Differential Equation:

It is a mathematical equation which involves the differential coefficients of the dependent variables.

Statistical Quantitative Techniques:

Statistical techniques are those techniques which are used in conducting the statistical enquiry concerning to certain Phenomenon. They include all the statistical methods beginning from the collection of data till interpretation of those collected data. Statistical techniques involve:

1. Collection of data:

One of the important statistical methods is collection of data. There are different methods for collecting primary and secondary data.

2. Measures of Central tendency, dispersion, skewness and Kurtosis

Measures of Central tendency is a method used for finding he average of a series while measures of dispersion used for finding out the variability in a series. Measures of Skewness measures asymmetry of a distribution while measures of Kurtosis measures the flatness of peakedness in a distribution.

3. Correlation and Regression Analysis:

Correlation is used to study the degree of relationship among two or more variables. On the other hand, regression technique is used to estimate the value of one variable for a given value of another.

4. Index Numbers:

Index numbers measure the fluctuations in various Phenomena like price, production etc over a period of time, They are described as economic barometres.

5. Time series Analysis:

Analysis of time series helps us to know the effect of factors which are responsible for changes:

6. Interpolation and Extrapolation:

Interpolation is the statistical technique of estimating under certain assumptions, the missing figures which may fall within the range of given figures. Extrapolation provides estimated figures outside the range of given data.

7. Statistical Quality Control

Statistical quality control is used for ensuring the quality of items manufactured. The variations in quality because of assignable causes and chance causes can be known with the help of this tool. Different control charts are used in controlling the quality of products.

8. Ratio Analysis:

Ratio analysis is used for analyzing financial statements of any business or industrial concerns which help to take appropriate decisions.

9. Probability Theory:

Theory of probability provides numerical values of the likely hood of the occurrence of events.

10. Testing of Hypothesis

Testing of hypothesis is an important statistical tool to judge the reliability of inferences drawn on the basis of sample studies.

Programming Techniques:

Programming techniques are also called operations research techniques. Programming techniques are model building techniques used by decision makers in modern times.

Programming techniques involve:

1.Linear Programming:

Linear programming technique is used in finding a solution for optimizing a given objective under certain constraints.

2. Queuing Theory:

Queuing theory deals with mathematical study of queues. It aims at minimizing cost of both servicing and waiting.

3. Game Theory:

Game theory is used to determine the optimum strategy in a competitive situation.

4. Decision Theory:

This is concerned with making sound decisions under conditions of certainty, risk and uncertainty.

5. Inventory Theory:

Inventory theory helps for optimizing the inventory levels. It focuses on minimizing cost associated with holding of inventories.

6. Net work programming:

It is a technique of planning, scheduling, controlling, monitoring and co-ordinating large and complex projects comprising of a number of activities and events. It serves as an instrument in resource allocation and adjustment of time and cost up to the optimum level. It includes CPM, PERT etc.

7. Simulation:

It is a technique of testing a model which resembles real life situations

8. Replacement Theory:

It is concerned with the problems of replacement of machines,etc due to their deteriorating efficiency or breakdown. It helps to determine the most economic replacement policy.

9. Non Linear Programming:

It is a programming technique which involves finding an optimum solution to a problem in which some or all variables are non-linear.

10. Sequencing:

Sequencing tool is used to determine a sequence in which given jobs should be performed by minimizing the total efforts.

11. Quadratic Programming:

Quadratic programming technique is designed to solve certain problems, the objective function of which takes the form of a quadratic equation.

12. Branch and Bound Technique

It is a recently developed technique. This is designed to solve the combinational problems of decision making where there are large numbers of feasible solutions. Problems of plant location, problems of determining minimum cost of production etc. are examples of combinational problems.

Functions of Quantitative Techniques:

The following are the important functions of quantitative techniques:

1. To facilitate the decision-making process
2. To provide tools for scientific research
3. To help in choosing an optimal strategy
4. To enable in proper deployment of resources
5. To help in minimizing costs
6. To help in minimizing the total processing time required for performing a set of jobs

Uses of Quantitate Techniques

Business and Industry

Quantitative techniques render valuable services in the field of business and industry. Today, all decisions in business and industry are made with the help of quantitative techniques.

Some important uses of quantitative techniques in the field of business and industry are given below:

Quantitative techniques of linear programming is used for optimal allocation of scarce resources in the problem of determining product mix

Inventory control techniques are useful in dividing when and how much items are to be purchase so as to maintain a balance between the cost of holding and cost of ordering the inventory

Quantitative techniques of CPM, and PERT helps in determining the earliest and the latest times for the events and activities of a project. This helps the management in proper deployment of resources.

Decision tree analysis and simulation technique help the management in taking the best possible course of action under the conditions of risks and uncertainty.

Queuing theory is used to minimize the cost of waiting and servicing of the customers in queues.

Replacement theory helps the management in determining the most economic replacement policy regarding replacement of equipment.

Limitations of Quantitative Techniques:

Even though the quantitative techniques are inevitable in decision-making process, they are not free from short comings. The following are the important limitations of quantitative techniques:

1. Quantitative techniques involves mathematical models, equations and other mathematical expressions
2. Quantitative techniques are based on number of assumptions. Therefore, due care must be ensured while using quantitative techniques, otherwise it will lead to

wrong conclusions.

3. Quantitative techniques are very expensive.
4. Quantitative techniques do not take into consideration intangible facts like skill, attitude etc.
5. Quantitative techniques are only tools for analysis and decision-making. They are not decisions itself.

Module II: Time Series and Index Number

Time series is the arrangement of data according to the time of occurrence. It helps to find our variations to the value of data due to changes in time.

Importance

1. It helps for understanding past behavior
2. It facilitates for forecasting and Planning
3. It facilitates comparison

Components of Time Series

1. Secular trend
2. Seasonal Variations
3. Cyclic Variations
4. Irregular Variations

Secular Trend

Trend may be defined as the changes over a long period of time. The significance of trend is greater when the period of time is very longer.

Following are the important method of measuring trend.

1. Graphic Method
2. Semi Average Method
3. Moving Average Method
4. Method of Least Squares

Seasonal Variations

Seasonal Variations are measured for one calendar year. It is the variations which occur some degree of regularity. For example climate conditions, social customs etc.

Cyclical Variations

Cyclical variations are those variation which occur on account of business cycle. They are Prosperity, Dectine, Depression and Recovery.

Irregular fluctuations

One changes of variable could not be predicted due to irregular movements. Irregular movements are like changes in technology, war, famines, flood etc.

Methods of Measuring Trend

Graphic method

It is otherwise known as free hand method. This is the simplest method of measuring trend. Under this method original data are plotted on the graph paper. The plotted points should be joined, we get a curve. A straight line should be drawn through the middle area of the curve. Such line will describe tendency of the data.

Semi Average Method

The whole data are divided in to two parts and average of these are to be calculated. The two averages are to be plotted in the graph. The two points plotted should be joined so as to get a straight line. This line is called the wardlive.

Method of Moving average

Under this method a series of successive average should be calculated from a series of values moving average may be calculated for 3,4,5,6 or 7years periods.

The moving average can be calculated as follows:

For example 3 years moving average will be $\frac{a+b+c,}{3}$, $\frac{b+c+d}{3}$, $\frac{c+d+e}{3}$ and so on.

Five years moving average = $\frac{a+b+c+d+e,}{5}$, $\frac{b+c+d+e+f}{5}$ and so on.

1. Compute 3 yearly moving average from the following data

Years:	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Sales(in 000 units)	55	47	59	151	79	36	45	72	83	89	102

Calculation of 3 yearly moving average

Year	Sales (in 000 units)	3 yearly moving total	3 yearly moving average
2002	55	----------	-----------
2003	47	----------	----------
2004	59	161	53.67
2005	151	257	85.67
2006	79	289	96.33
2007	36	216	58.67
2008	45	160	63.33
2009	72	153	51
2010	83	200	66.67
2011	89	244	81.33
2012	102	277	91.33

2. Calculate 5 yearly moving averages

Years:	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
income (in ‘000’)	161	127	152	143	144	167	182	179	152	163	159

Solution

Year	**Income (in000)**	**Five yearly moving total**	**Five yearly moving average**
2000	161	----------	-----------

2001	127	----------	----------
2002	152	727	145.4
2003	143	733	146.6
2004	144	788	157.6
2005	167	815	163
2006	182	824	164.8
2007	179	843	168.6
2008	152	835	167
2009	163	----------	-----------
2010	159	----------	----------

Calculation of moving average for every periods

1) Calculate the six year moving average

Years:	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Demand (intones)	105	120	115	110	100	130	135	160	155	140	145

Solution

Year	Demand	6 years moving total	6 years moving average	Centered 6 years moving total	Centered 6 year moving average
2000	105	-------	-------	-------	-------
2001	120	--------	--------	--------	--------
2002	115	---------	---------	---------	---------
2003	110	680	113.3	231.6	115.8
2004	100	710	118.3	243.3	121.65
2005	130	750	125	256.67	128.34
2006	135	790	131.67	268.34	134.17
2007	160	820	136.67	280.84	140.42
2008	155	865	144.17		
2009	140				
2010	145				

Method of Least Squares

This is a popular method of obtaining trend line. The trend line obtained through this method is called line of best fit.

One trend line is represented as $y = a + bx$

The value of **a** and **b** can be ascertained by solving the following two normal equations.

$\sum y = Na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$

Where **x** represents the time, **y** represents the value, **a** and **b** are constant and **N** represent total number.

When the middle year is taken as the origin, then $\sum x = 0$, then normal equation would be

$\sum xy = Na$

$\sum xy = b\sum x^2$

Hence $a = \frac{\sum xy}{\sum x^2}$

1. Following are the data related with the output of a factory for 7 years

Years:	2006	2007	2008	2009	2010	2011	2012
Output (in tones)	47	64	77	88	97	109	113

Calculate the trend values through the method of least squares and also forecast the production 2013 and 2015.

Solution

Yeart	Productiony	x (t – 2009)	xy	x^2
2006	47	-3	-141	9
2007	64	-2	-128	4
2008	77	-1	-77	1
2009	88	0	0	0
2010	97	1	97	1
2011	109	2	218	4
2012	113	3	339	9
	595	0	308	28

Here ∑x = 0

A = = $\frac{\sum y}{n}$ $\frac{595}{7}$ $= 85$

b = $\frac{\sum xy}{\sum x^2} = \frac{308}{28}$ = 11

y = a + bx

2006 - 85 + 11 x -3 = 52

2007 - 85 + 11 x -2 = 63

2008 - 85 + 11 x -1 = 74

2009 - 85 + 11 x 0 = 85

2010 - 85 + 11 x 1 = 96

2011 - 85 + 11 x 2 = 107

2012 - 85 + 11 x 3 = 118

Production in 2013

= 85 x 11 x 4 = 129 tonns

Production in 2015

= 85 x 11 x 6 = 151 tonns

Index Numbers

Index numbers is a statistical device for measuring the changes in group of related variables over a period of time.

Uses or Importance of index numbers.

1. Index numbers measure trend values.
2. Index numbers facilitate for policy decisions.
3. Index numbers help in comparing the standard of living.
4. It measures changes in price level.

5. Index numbers are economic barometers. The condition of the economy of a country to be known through construction of index numbers for different periods with regard to employment, literacy, agriculture industry, economics etc. Hence it can be termed as economic barometers.

Limitations

1. Index numbers are only approximate indicator.
2. All index numbers are not good for all purposes.
3. Index numbers are liable to be unissued.
4. Index numbers are specilised average and limitations of average also applicable to index numbers.

Problems or Difficulties in the construction of index numbers

1. Purpose of the index.
2. Selection of the lease period.
3. Selection of items.
4. Selection of an average
5. Selection of weights
6. Selection of appropriate source of data
7. Selection of suitable formula.

Methods of constructing index numbers

1. Unweighted index numbers.

2. Weighted index numbers.

Unweighted or Simple index numbers

Simple index numbers are those index numbers in which all items are treated as equally. Simple aggregate and simple average price relatives are the unweighted index numbers.

1. Simple Aggregate method

$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$

P01 = index number

P1 = Price for the current year

P1 = Price for the base year.

2. Simple Average Price Relative Method

$P_{01} = \frac{\Sigma I}{n}$

$I = \frac{p_1}{p_0} \times 100$, each items can be calculated

Weighted index numbers

In this method quantity consumed is also taken into account. Such index are-

1. Weighted aggregate method
2. Weighted Average of price relatives

Weighted aggregate method

This method is based on the weight of the prices of the selected commodities.Following are the commonly used methods:

1. Laspeyre's Method
2. Paasche's Method
3. Bowley-Dorbish Method
4. Fishers ideal method
5. Kelly's Methods

Laspeyre's Method

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

p_1 = Price of the current year

q_0 = Quantity of the base year

p_0 = Price of the base year

Paasche's Method

$$P_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

q_1 = Quantity of the current year

Fishers Ideal Method

$$P_{01} = \sqrt{L \times P} \times 100$$

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_1} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

L = Laspeyres method

P = Paasche's Method

Bowley-Doribish Method

$$P_{01} = \frac{L+P}{2}$$

Kelly's Method

$$P_{01} = \frac{\sum p_1 q}{\sum p_0 q} \times 100$$

$$q = \frac{q_0+q_1}{2}$$

Weighted Average Price Relative Method

Index number = $\frac{\sum IV}{\sum v}$

V = Weight

$$I = \frac{P_1}{P_0} \times 100$$

1. Construct index numbers for 2012 on the basis of the price of 2010

Commodities	Price in 2010	Price in 2012
A	115	130
B	72	89
C	54	75
D	60	72
E	80	105

Solution

Commodities	P_0	P_1
A	115	130
B	72	89
C	54	75
D	60	72
E	80	105
	381	**471**

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

$$P_{01} = \frac{471}{381} \times 100 = 123.62$$

2. Calculate simple index number by average relative method.

Items	Price of the base year	Price of the current year
A	5	7
B	10	12
C	15	25
D	20	18
E	8	9

Solution

Items	P_0	P_1	ie $\frac{p_1}{p_0} \times 100$
A	5	7	140
B	10	12	120
C	15	25	166.7
D	20	18	90
E	8	9	112.5
			629.2

Index number = $\frac{\sum I}{n}$ = $\frac{629.6}{5} = 125.84$

3. Following are the data related with the prices and quantities consumed for 2010 and2012.

Commodity	2010		2012	
	Price	Quantity	Price	Quantity
Rice	5	15	7	12
Wheat	4	5	6	4
Sugar	7	4	9	3
Tea	52	2	55	2

Construct price index numbers by

1. Laspeyre’s method
2. Paasche’s method
3. Bowly’s – Dorbish method
4. Fisher’s method

Solution

Commodity	p_0	q_0	p_1	q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
Rice	5	15	7	12	105	75	84	60
Wheat	4	5	6	4	30	20	24	16
Sugar	7	4	9	3	36	28	27	21
Tea	12	2	55	2	110	104	110	104
					281	**227**	**245**	**201**

1. Laspeyre’s Method

$$P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 \qquad = \frac{281}{227} \times 100$$

$$= \underline{\underline{123.79}}$$

2. Paascne’s method

$$P_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 \qquad = \frac{245}{201} \times 100$$

$= \underline{\underline{121.89}}$

3. Bowley – Dorbish Method

$P_{01} = \frac{L+P}{2}$ $= \frac{123.79+121.89}{2}$

$= \underline{\underline{122.84}}$

4. Fisher's Method

$P_{01} = \sqrt{L \times P}$ $= \sqrt{123.79 \times 121.89}$

$= 122.84$

4) Calculate index number of price for 2012 on the basis of 2010, from the data given below:

Commodities	Weight	Price 2010	Price2012
A	40	16	20
B	25	40	60
C	5	2	2
D	20	5	6
E	10	2	1

Solution

$$Price\ Index\ Number = \frac{\sum IV}{\sum v}$$

Commodities	V	p_0	p_1	i.e. $\frac{p1}{p0}$ x 100	IV
A	40	16	20	125	5000
B	25	40	60	150	3750
C	5	2	2	100	500
D	20	5	6	120	2400
E	10	2	1	50	500
	100				12150

$$Index\ Number = \frac{12150}{100}$$

5. Construct Price Index

Commodities	Index	Weight
A	350	5
B	200	2
C	240	3
D	150	1
E	250	2

Solution

Commodities	V	I	IV
A	5	350	1750
B	2	200	400
C	3	240	720
D	1	150	150
E	2	250	500
	13		**3520**

$$Price\ Index\ Number = \frac{\sum IV}{\sum v} = \frac{3520}{13} = \underline{\underline{270.77}}$$

Consumer Price index number of cost of Living index number or Retail Price index number

Consumer Price index number is also known as copy of Living Index number or Retails Price index number. It is the ration of the monetary expenditures of an individual which secure him the standard of living or total utility in two situations differing only in respect of prices. It represents the average change in prices over a period of time, paid by the consumer for goods and services.

Steps in the construction of Consumer Price Index

1. Determination of the class people for whom the index

number is to construct.

2. Selection of Basic period
3. Conducting family budget enquiry
4. Obtaining price quotation
5. Selecting proper weights
6. Selection of suitable methods for constructing index.

Methods of Constructing Consumer Price Index Number

1. Aggregate Expenditure Method

Cost of living Index number = $P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$

2. **Family Budget Method or Average Relative Method**

Cost of Living Index = $\frac{\Sigma IV}{\Sigma v}$

1. Find cost of Living index

	Food	Rent	Clothes	Fuel	Miscellanious
Expenses on	35%	15%	20%	10%	25%
Price 2010	150	30	75	25	40
Price 2012	145	30	65	23	45

What changes the cost of living of 2012 as compare to 2010?

Solution

Expenses	V	p_0	p_1	I	IV
Food	35	150	145	96.67	3383.45
Rent	15	30	30	100	1500
Cloth	20	75	65	86.67	1733
Fuel	10	25	23	92	920
Misc.	20	40	45	112.50	2250
					9786.85

Cost of Living Index = $\frac{\sum IV}{\sum v} = \frac{9786.85}{100} =$ 97.87

Module III: Correlation and Regression Analysis

In practice, we may come across with lot of situations which need statistical analysis of either one or more variables. The data concerned with one variable only is called univariate data. For Example: Price, income, demand, production, weight, height marks etc are concerned with one variable only. The analysis of such data is called univariate analysis.

The data concerned with two variables are called bivariate data. For example: rainfall and agriculture; income and consumption; price and demand; height and weight etc. The analysis of these two sets of data is called bivariate analysis.

The date concerned with three or more variables are called multivariate date. For example: agricultural production is influenced by rainfall, quality of soil, fertilizer etc.

The statistical technique which can be used to study the relationship between two or more variables is called correlation analysis.

Definition:

Two or more variables are said to be correlated if the change in one variable results in a corresponding change in the other variable.

According to Simpson and Kafka, “Correlation analysis deals with the association between two or more variables”.

Lunchou defines, “Correlation analysis attempts to determine the degree of relationship between variables”.

Boddington states that “Whenever some definite connection exists between two or more groups or classes of series of data, there is said to be correlation.”

In nut shell, correlation analysis is an analysis which helps to determine the degree of relationship exists between two or more variables.

Correlation Coefficient:

Correlation analysis is actually an attempt to find a numerical value to express the extent of relationship exists between two or more variables. The numerical measurement showing the degree of correlation between two or more variables is called correlation coefficient. Correlation coefficient ranges between -1 and +1.

Significance of Correlation Analysis

1. Correlation analysis is of immense use in practical life because of the following reasons:
2. Correlation analysis helps us to find a single figure to measure the degree of relationship exists between the variables. Correlation analysis helps to understand the

economic behavior.

3. Correlation analysis enables the business executives to estimate cost, price and other variables.
4. Correlation analysis can be used as a basis for the study of regression. Once we know that two variables are closely related, we can estimate the value of one variable if the value of other is known.
5. Correlation analysis helps to reduce the range of uncertainty associated with decision making. The prediction based on correlation analysis is always near to reality.
6. It helps to know whether the correlation is significant or not. This is possible by comparing the correlation co-efficient with 6PE. It 'r' is more than 6 PE, the correlation is significant.

Classification of Correlation

Correlation can be classified in different ways. The following are the most important classifications

1. Positive and Negative correlation
2. Simple, partial and multiple correlation
3. Linear and Non-linear correlation

Positive and Negative Correlation

Positive Correlation

When the variables are varying in the same direction, it is called positive correlation. In other words, if an increase in the value of one variable is accompanied by an increase in the value of other variable or if a decrease in the value of one variable is accompanied by a decree se in the value of other variable, it is called positive correlation.

Eg: 1)	A:	10	20	30	40	50
	B:	80	100	150	170	200
2)	X:	78	60	52	46	38
	Y:	20	18	14	10	5

Negative Correlation:

When the variables are moving in opposite direction, it is called negative correlation. In other words, if an increase in the value of one variable is accompanied by a decrease in the value of other variable or if a decrease in the value of one variable is accompanied by an increase in the value of other variable, it is called negative correlation.

Eg: 1)	A:	5	10	15	20	25
	B:	16	10	8	6	2
2)	X:	40	32	25	20	10

Y:	2	3	5	8	12

Simple, Partial and Multiple correlation

Simple Correlation

In a correlation analysis, if only two variables are studied it is called simple correlation. Eg. the study of the relationship between price & demand, of a product or price and supply of a product is a problem of simple correlation.

Multiple correlation

In a correlation analysis, if three or more variables are studied simultaneously, it is called multiple correlation. For example, when we study the relationship between the yield of rice with both rainfall and fertilizer together, it is a problem of multiple correlation.

Partial correlation

In a correlation analysis, we recognize more than two variable, but consider one dependent variable and one independent variable and keeping the other Independent variables as constant. For example yield of rice is influenced b the amount of rainfall and the amount of fertilizer used. But if we study the correlation between yield of rice and the amount of rainfall by keeping the amount of fertilizers used as constant, it is a problem of partial correlation.

Linear and Non-linear correlation

Linear Correlation

In a correlation analysis, if the ratio of change between the two sets of variables is same, then it is called linear correlation.

For example when 10% increase in one variable is accompanied by 10% increase in the other variable, it is the problem of linear correlation.

X:	10	15	30	60
Y:	50	75	150	300

Here the ratio of change between X and Y is the same. When we plot the data in graph paper, all the plotted points would fall on a straight line.

Non-linear correlation

In a correlation analysis if the amount of change in one variable does not bring the same ratio of change in the other variable, it is called non linear correlation.

X:	2	4	6	10	15
Y:	8	10	18	22	26

Here the change in the value of X does not being the same proportionate change in the value of Y.

This is the problem of non-linear correlation, when we plot the data on a graph paper, the plotted points would not fall on a straight line.

Degrees of correlation

Correlation exists in various degrees

Perfect positive correlation

If an increase in the value of one variable is followed by the same proportion of increase in other related variable or if a decrease in the value of one variable is followed by the same proportion of decrease in other related variable, it is perfect positive correlation. eg: if 10% rise in price of a commodity results in 10% rise in its supply, the correlation is perfectly positive. Similarly, if 5% full in price results in 5% fall in supply, the correlation is perfectly positive.

Perfect Negative correlation

If an increase in the value of one variable is followed by the same proportion of decrease in other related variable or if a decrease in the value of one variable is followed by the same proportion of increase in other related variably it is Perfect Negative Correlation. For example if 10% rise in price results in 10% fall in its demand the correlation is perfectly negative. Similarly if 5% fall in price results in 5% increase in demand, the correlation is perfectly negative.

Limited Degree of Positive correlation:

When an increase in the value of one variable is followed by a non -proportional increase in other related variable, or when a decrease in the value of one variable is followed by a nonproportional decrease in other related variable, it is called limited degree of positive correlation.

For example, if 10% rise in price of a commodity results in 5% rise in its supply, it is limited degree of positive correlation. Similarly if 10% fall in price of a commodity results in 5% fall in its supply, it is limited degree of positive correlation.

Limited degree of Negative correlation

When an increase in the value of one variable is followed by a non-proportional decrease in other related variable, or when a decrease in the value of one variable is followed by anonproportional increase in other related variable, it is called limited degree of negative correlation.

For example, if 10% rise in price results in 5% fall in its demand, it is limited degree of negative correlation. Similarly, if 5% fall in price results in 10% increase in demand, it is limited degree of negative correlation.

Zero Correlation (Zero Degree correlation)

If there is no correlation between variables it is called zero correlation. In other words, if the values of one variable cannot be associated with the values of the other variable, it is zero correlation.

Methods of measuring correlation

Correlation between 2 variables can be measured by graphic methods and algebraic methods.

I Graphic Methods

a. Scatter Diagram
b. Correlation graph

II Algebraic methods (Mathematical methods or statistical methods or Co-efficient of correlationmethods):

a. Karl Pearson’s Co-efficient of correlation
b. Spear mans Rank correlation method
c. Concurrent deviation method

Scatter Diagram

This is the simplest method for ascertaining the correlation between variables. Under this method all the values of the two variable are plotted in a chart in the form of dots. Therefore, it is also known as dot chart. By observing the scatter of the various dots, we can form an idea that whether the variables are related or not.

A scatter diagram indicates the direction of correlation and tells us how closely the two variables under study are related. The greater the scatter of the dots, the lower is the relationship.

Figure 1 :Perfect Positive Correlation

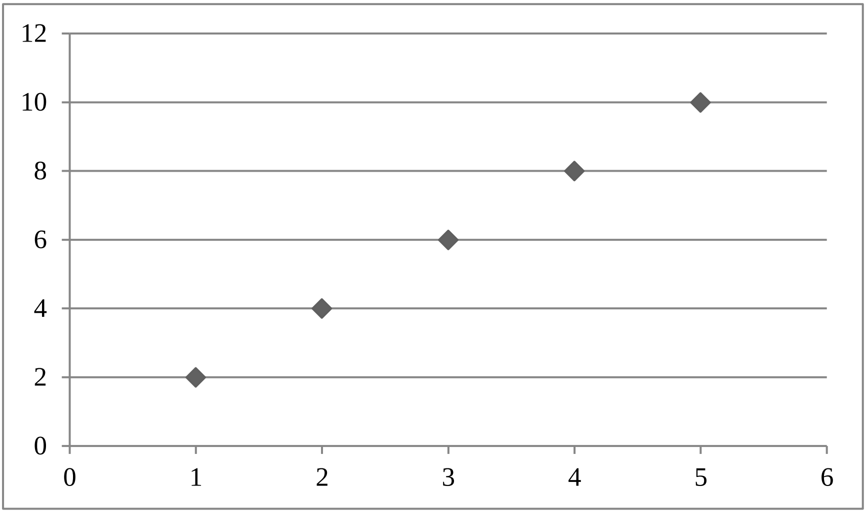
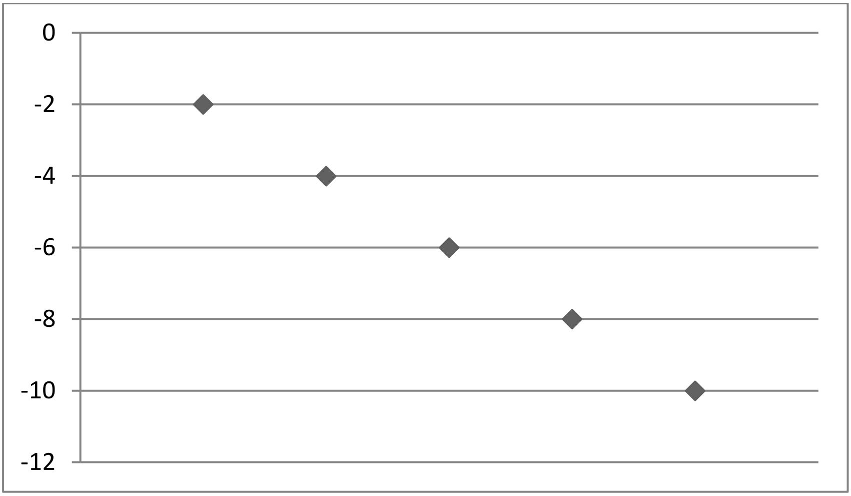


Figure 1 :Perfect Negative Correlation



Merits of Scatter Diagram method

1. It is a simple method of studying correlation between variables.
2. It is a non-mathematical method of studying correlation between the variables. It does not require any mathematical calculations.
3. It is very easy to understand. It gives an idea about the correlation between variables even to a layman.

4. It is not influenced by the size of extreme items.
5. Making a scatter diagram is, usually, the first step in investigating the relationship between two variables.

Demerits of Scatter diagram method

It gives only a rough idea about the correlation between variables.

The numerical measurement of correlation co-efficient cannot be calculated under this method.

It is not possible to establish the exact degree of relationship between the variables.

Correlation graph Method

Under correlation graph method the individual values of the two variables are plotted on a graph paper. Then dots relating to these variables are joined separately so as to get two curves. By examining the direction and closeness of the two curves, we can infer whether the variables are related or not. If both the curves are moving in the same direction(either upward or downward) correlation is said to be positive. If the curves are moving in the opposite directions, correlation is said to be negative.

Merits of Correlation Graph Method

1. This is a simple method of studying relationship between the variable
2. This does not require mathematical calculations.

3. This method is very easy to understand

Demerits of correlation graph method:

1. A numerical value of correlation cannot be calculated.
2. It is only a pictorial presentation of the relationship between variables.
3. It is not possible to establish the exact degree of relationship between the variables.

Karl Pearson's Co-efficient of Correlation

Karl Pearson's Coefficient of Correlation is the most popular method among the algebraic methods for measuring correlation. This method was developed by Prof. Karl Pearson in 1896. It is also called product moment correlation coefficient.

Pearson's coefficient of correlation is defined as the ratio of the covariance between X and Y to the product of their standard deviations. This is denoted by 'r' or r_{xy}

$$r = \frac{\text{Covariance of X and Y}}{(\text{SD of X}) \text{ x } (\text{SD of Y})}$$

Interpretation of Co-efficient of Correlation

Pearson's Co-efficient of correlation always lies between +1 and - The following general rules will help to interpret the Co-efficient of correlation:

1. When r - +1, It means there is perfect positive relationship

between variables.

2. When r = -1, it means there is perfect negative relationship between variables.
3. When r = 0, it means there is no relationship between the variables.
4. When ‘r’ is closer to +1, it means there is high degree of positive correlation between variables.
5. When ‘r’ is closer to – 1, it means there is high degree of negative correlation between variables.
6. When ‘r’ is closer to ‘O’, it means there is less relationship between variables.

Properties of Pearson’s Co-efficient of Correlation

1. If there is correlation between variables, the Co-efficient of correlation lies between +1 and -1.
2. If there is no correlation, the coefficient of correlation is denoted by zero (ie r=0)
3. It measures the degree and direction of change
4. If simply measures the correlation and does not help to predict cansation.
5. It is the geometric mean of two regression co-efficients.

i.e $r = \sqrt{b\text{xy} \,.\, byx}$

Computation of Pearson's Co-efficient of correlation:

Pearson's correlation co-efficient can be computed in different ways. They are:

1. Arithmetic mean method
2. Assumed mean method
3. Direct method

Arithmetic mean method:-

Under arithmetic mean method, co-efficient of correlation is calculated by taking actual mean.

$$r = \frac{\Sigma(x-\overline{x})(y-\overline{y})}{\sqrt{\Sigma(x-\overline{x})2\ \Sigma(y-\overline{y})2}}$$

or $$r = \frac{\Sigma xy}{\sqrt{\Sigma x2\ \Sigma y2}}$$

Calculate Pearson's co-efficient of correlation between age and playing habits of students:

Age:	20	21	22	23	24	25
No. of students	500	400	300	240	200	160
Regular players	400	300	180	96	60	24

Let X = Age and Y = Percentage of regular players

Percentage of regular players can be calculated as follows:-

400 x 100 = 80; 300 x 100 = 75; 180 x 100 = 60; 96 x 100 = 40 ,

500 400 300 240

$\frac{60\ \text{x}}{20}$ 100 = 30; and $\frac{24\ \text{x}}{160}$ 100 = 15

Pearson's Coefficient of Correlation (r) = $\frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})2\ \Sigma(y-\bar{y})2}}$

Computation of Pearson's Coefficient of correlation						
Age x	% of Regular Player y	(x-22.5)	(y-50)	(x-$\bar{x}$) (y-$\bar{y}$)	(x-$\bar{x}$)^2	(y-$\bar{y}$)^2
20	80	-2.5	30	-75.0	6.25	900
21	75	-1.5	25	-37.5	2.25	625
22	60	-0.5	10	- 5.0	0.25	100
23	40	0.5	-10	- 5.0	0.25	100
24	30	1.5	-20	-30.0	2.25	400
25	15	2.5	-35	-87.5	6.25	1225
135	300			-240	17.50	3350

$$\overline{x} = \frac{\Sigma x}{N} = \frac{135}{6} = 22.5$$

$$\overline{y} = \frac{\Sigma \mathrm{y}}{N} = \frac{300}{6} = 50$$

r = $\frac{-240}{\sqrt{17.5\times3350}} = \frac{-240}{\sqrt{58625}} = \frac{-240}{\sqrt{242.126}} = -0.9912$

Assumed mean method:

Under assumed mean method, correlation coefficient is calculated by taking assumean only.

$$\frac{N\sum dxdy - (\sum dx)(\sum dy)}{\sqrt{N\sum dx^2 - (\sum dx)^2} \times \sqrt{N\sum dy^2 - (\sum dy)^2}}$$

Where dx = deviations of X from its assumed mean; dy= deviations of y from its assumed mean Find out coefficient of correlation between size and defect in quality of shoes:

Size : 15-16 16-17 17-18 18-19
19-20 20-21

No. of shoes}
Produced : 200 270 340 360
400 300

No. of defectives: 150 162 170 180 180
114 Let x = size (ie mid-values) y = percentage of defectives

x values are 15.5 , 16.5, 17.5, 18.5, 19.5 and 20.5

y values are 75, 60, 50, 50, 45 and 38

Take assumed mean: x = 17.5 and y = 50

Computation of Pearson's Co efficient of Correlation						
x	y	dx	dy	dxdy	dx^2	dy^2
15.5	75	-2	25	-50	4	625
16.5	60	-1	10	-10	1	100
17.5	50	0	0	0	0	0
18.5	50	1	0	0	1	0
19.5	45	2	-5	-10	4	25
20.5	38	3	-12	-36	9	144
		∑dx 3	∑dy 18	∑dxdy = -106	∑dx^2 = 19	∑dy^2 = 894

$$r = \frac{N\sum dxdy - (\sum dx)(\sum dy)}{\sqrt{N\sum dx^2 - (\sum dx)^2} \times \sqrt{N\sum dy^2 - (\sum dy)^2}}$$

$$r = \frac{(6 \times -106) - (3 \times 18)}{\sqrt{(6 \times 19) - 3^2} \times \sqrt{(6 \times 894) - 18^2}}$$

$$\frac{-636 - 54}{\sqrt{114 - 9} \times \sqrt{5364 - 324}}$$

$= \frac{-690}{\sqrt{105} \times \sqrt{5040}} = \frac{-690}{727.46}$ = -0.9485

Direct Method:

Under direct method, coefficient of correlation is calculated without taking actual mean or assumed mean

$$r = \frac{N\sum xy - (\sum x)(\sum y)}{\sqrt{N\sum x^2 - (\sum x)^2} \times \sqrt{N\sum y^2 - (\sum y)^2}}$$

From the following data, compute Pearson's correlation coefficient:

Price :	10	12	14	15	19
Demand (Qty)	40	41	48	60	50

Let us take price = x and demand = y

Computation of Pearson's Coefficient of Correlation				
Price (x)	Demand (y)	xy	x^2	y^2
10	40	400	100	1600
12	41	492	144	1681
14	48	672	196	2304
15	60	900	225	3600
19	50	950	361	2500

$\sum x = 70$	$\sum y = 239$	$\sum xy = 3414$	$\sum x^2$ 1026	$\sum y^2$=11685

$$r = \frac{N\sum xy - (\sum x)(\sum y)}{\sqrt{N\sum x^2 - (\sum x)^2} \times \sqrt{N\sum y^2 - (\sum y)^2}} \quad =$$

$$r = \frac{(5 \times 3414) - (70 \times 239)}{\sqrt{(5 \times 1026) - 70^2} \times \sqrt{(5 \times 11685) - 239^2}}$$

$$r = \frac{17070 - 16730}{\sqrt{230} \times \sqrt{1304}} \quad = 0.621$$

Probable Error and Coefficient of Correlation

Probable error (PE) of the Co-efficient of correlation is a statistical device which measures the reliability and dependability of the value of co-efficient of correlation.

Probable Error = $\frac{2}{3}$ standard error

= 0.6745 x standard error

Standard Error (SE) = $\frac{1-r^2}{\sqrt{n}}$

$$\therefore PE = 0.6745 \times \frac{1-r^2}{\sqrt{n}}$$

If the value of coefficient of correlation (r) is less than the PE, then there is no evidence of correlation.

If the value of ‘r’ is more than 6 times of PE, the correlation is certain and significant.

By adding and submitting PE from coefficient of correlation, we can find out the upper and lower limits within which the population coefficient of correlation may be expected to lie.

Uses of PE:

1. PE is used to determine the limits within which the population coefficient of correlation may be expected to lie.
2. It can be used to test whether the value of correlation coefficient of a sample is significant with that of the population

Qn: 1. If r = 0.6 and N = 64, find out the PE and SE of the correlation coefficient. Also determine the limits of population correlation coefficient.

Sol: r = 0.6

N=64

PE = 0.6745 SE

$$SE = \frac{1-r^2}{\sqrt{n}} \qquad = \frac{1-(0.6)^2}{\sqrt{64}} = \frac{0.64}{8} = 0.08$$

P.E = 0.6745 × 0.08

= 0.05396

Limits of population Correlation coefficient = $r \pm PE$

$= 0.6 \pm 0.05396$

= <u>0.54604 to 0.6540</u>

Qn. 2 r and PE have values 0.9 and 0.04 for two series. Find n.

Sol: PE = 0.04

$$= 0.6745\ \frac{1-r^2}{\sqrt{n}} = 0.04$$

$$\frac{1-09^2}{\sqrt{n}} = \frac{0.04}{0.6745}$$

$$= \frac{1-0.81}{\sqrt{n}} = 0.0593$$

$$= \frac{0.19}{\sqrt{n}} = 0.0593$$

$$= 0.0593 \times \sqrt{n = 0.19}$$

$$= \sqrt{n} = \frac{0.19}{0.0593}$$

= N = 3.2^2 = 10. 266

N = 10

Coefficient of Determination

One very convenient and useful way of interpreting the value of coefficient of correlation is the use of the square of coefficient of correlation. The square of coefficient of correlation is called coefficient of determination.

Coefficient of determination = r^2

Coefficient of determination is the ratio of the explained variance to the total variance.

For example, suppose the value of r = 0.9, then r^2 = 0.81=81%

This means that 81% of the variation in the dependent variable has been explained by (determined by) the independent variable. Here 19% of the variation in the dependent variable has not been explained by the independent variable. Therefore, this 19% is called coefficient of non-determination.

Coefficient of non-determination (K^2) = $1 – r^2$

K^2 = 1- coefficient of determination

Qn: Calculate coefficient of determination and non-determination if coefficient of correlation is 0.8

Sol:- r =0.8

Coefficient of determination $= r^2$

$= 0.8^2 =0.64 =$ 64%

Co efficient of non-determination $= 1 – r^2$

$= 1- 0.64$

$= 0.36$

= <u>36%</u>

Merits of Pearson's Coefficient of Correlation:-

1. This is the most widely used algebraic method to measure coefficient of correlation.
2. It gives a numerical value to express the relationship

between variables

3. It gives both direction and degree of relationship between variables
4. It can be used for further algebraic treatment such as coefficient of determination coefficient of non-determination etc.
5. It gives a single figure to explain the accurate degree of correlation between two variables

Demerits of Pearson's Coefficient of correlation

1. It is very difficult to compute the value of coefficient of correlation.
2. It is very difficult to understand
3. It requires complicated mathematical calculations
4. It takes more time
5. It is unduly affected by extreme items
6. It assumes a linear relationship between the variables. But in real life situation, it may not be so.

Spearman's Rank Correlation Method

Pearson's coefficient of correlation method is applicable when variables are measured in quantitative form. But there were many cases where measurement is not possible because of the qualitative nature of the variable. For example, we cannot measure the beauty, morality, intelligence, honesty etc in quantitative terms. However it is possible to rank these qualitative characteristics in some order.

The correlation coefficient obtained from ranks of the variables instead of their quantitative measurement is called rank correlation. This was developed by Charles Edward Spearman in 1904. Spearman’s coefficient correlation (R) = $1 - \frac{6\sum D^2}{N^3 - N}$

Where D = difference of ranks between the two variables

N = number of pairs

Qn: Find the rank correlation coefficient between poverty and overcrowding from the information given below:

Town: A B C D E F G H I
J

Poverty: 17 13 15 16 6 11 14 9
7 12

Over crowing:36 46 35 24 12 18 27 22 2
8

Sol: Here ranks are not given. Hence we have to assign ranks

$$R = 1 - \frac{6\sum D^2}{N^3 - N}$$

N = 10

Computation of rank correlation Co-efficient

Town	Poverty	Over crowding	R_1	R_2	D	D^2
A	17	36	1	2	1	1
B	13	46	5	1	4	16
C	15	35	3	3	0	0
D	16	24	2	5	3	9
E	6	12	10	8	2	4
F	11	18	7	7	0	0
G	14	27	4	4	0	0
H	9	22	8	6	2	4
I	7	2	9	10	1	1
J	12	8	6	9	3	9
$\sum D^2$						44

$$R = 1 - \frac{6\times 44}{10^3 - 10} = 1 - \frac{264}{990} = 1- 0.2667$$

$$=0.7333$$

Qn:- Following were the ranks given by three judges in a beauty context. Determine which pair of judges has the nearest approach to Common tastes in beauty.

Judge I:	1	6	5	10	3	2	4	9	7	8
Judge I:	3	5	8	4	7	10	2	1	6	9
Judge I:	6	4	9	8	1	2	3	10	5	7

$R \quad = 1 - \frac{6\sum D^2}{N^3 - N} \qquad N= 10$

Computation of Spearman's Rank Correlation Coefficient								
Judge I (R_1)	Judge II (R_2)	Judge III (R_3)	R_1-R_2 (D_1)	R2-R3 (D_2)	R1-R3 (D_3)	D_1^2	D_2^2	D_3^2
1	3	6	2	3	5	4	9	25
6	5	4	1	1	2	1	1	4
5	8	9	3	1	4	9	1	16
10	4	8	6	4	2	36	16	4

3	7	1	4	6	2	16	36	4
2	10	2	8	8	0	64	64	0
4	2	3	2	1	1	4	1	1
9	1	10	8	9	1	64	81	1
7	6	5	1	1	2	1	1	4
8	9	7	1	2	1	1	4	1
$\sum D^2$						200	214	60

R = $1 - \frac{6\sum D^2}{N^3 - N}$

Rank correlation coefficient between I & II =1 - $\frac{6\times200}{10^3-10}$

$= 1 - \frac{1200}{990}$

= 1- 1.2121

= - $\underline{0.2121}$

Rank correlation Coefficient between II & III judges= $1 - \frac{6\times214}{10^3-10}$

$= 1 - \frac{1284}{990}$

=-0.297

Rank correlation coefficient between I& II judges = $1 - \frac{6\times60}{10^3-10}$

= $1 - \frac{360}{990}$

$= 1- 0.364$

$= \underline{0.636}$

The rank correlation coefficient in case of I& III judges is greater than the other two pairs. Therefore, judges I & III have highest similarity of thought and have the nearest approach to common taste in beauty.

Qn: The Co-efficient of rank correlation of the marks obtained by 10 students in statistics & English was 0.2. It was later discovered that the difference in ranks of one of the students was wrongly takes as 7 instead of 9 find the correct result.

R $= 0.2$

$= 1 - \frac{6\sum D^2}{N^3 - N} = 0.2$

$= \frac{1-0.2}{1} = \frac{6\sum D^2}{10^3 - 10}$

$= \frac{0.8}{1} = \frac{6\sum D^2}{990}$

$= 6\sum D^2 = 990 \times 0.8 = 792$

Correct $\sum D^2 = \frac{792}{6} = 132 - 7^2 {}^+ 9^2 = \underline{164}$

Correct R $= 1 - \frac{6\sum D^2}{N^3 - N} = 1 - \frac{6 \times 164}{10^3 - 10}$

$= 1 - \frac{984}{990} = 1 - 0.9939$

$= \underline{\underline{0.0061}}$

Qn: The coefficient of rank correlation between marks in English and maths obtained by a group students is 0.8. If the sum of the squares of the difference in ranks is given to be 33, find the number of students in the group.

Sol: $R = 1 - \frac{6\sum D^2}{N^3 - N} = 0.8$

ie, $= 1 - \frac{6\times 33}{N^3 - N} = 0.8$

$= 1 - 0.8 = \frac{6\times 33}{N^3 - N}$

$= 0.2\times (N^3 - N) = 198$

$= N^3 - N = \frac{198}{0.2} = 990$

N = 10

Computation of Rank Correlation Coefficient when Ranks are Equal

There may be chances of obtaining same rank for two or more items. In such a situation, it is required to give average rank for all. Such items. For example, if two observations got 4th rank, each of those observations should be given the rank 4.5 (i.e$\frac{4+5}{2}$ =4.5)

Suppose 4 observations got 6th rank, here we have to assign the rank, 7.5 (ie.$\frac{6+7+8+9}{4}$) to each of the 4 observations.

When there is an equal rank, we have to apply the following formula to compute rank correlation coefficient:-

$$R = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \ldots\ldots\right]}{N^3 - N}$$

Where D – Difference of rank in the two series

N - Total number of pairs

m - Number of times each rank repeats

Qn:- Obtain rank correlation co-efficient for the data:-

X : 68 64 75 50 64 80 75 40
55 64

Y: 62 58 68 45 81 60 68 48
50 70

Here, ranks are not given we have to assign ranks Further; this is the case of equal ranks.

$$\therefore R = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \ldots\ldots\right]}{N^3 - N}$$

Computation of rank correlation coefficient					
x	y	R_1	R_2	$D(R_1\text{-}R_2)$	D^2
68	62	4	5	1	1

64	58	6	7	1	1
75	68	2.5	3.5	1	1
50	45	9	10	1	1
54	81	6	1	5	25
80	60	1	6	5	25
75	68	2.5	3.5	1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
				ΣD^2	72

$$R= = 1-\frac{6\left[72+\frac{1}{12}(2^3-2)+\frac{1}{12}(3^3-3)+\frac{1}{12}(2^3-2)\right]}{10^3-10}$$

$$= 1-\frac{6\left[72+\frac{1}{12}+2+\frac{1}{12}\right]}{10^3-10}$$

$$= 1-\frac{6\times[72+3]}{990}$$

$$=1-\frac{6\times 75}{990}$$

$$= 1-\frac{450}{990} = 1- 0.4545$$

$$= 0.5455$$

Merits of Rank Correlation method

1. Rank correlation coefficient is only an approximate measure as the actual values are not used for calculations
2. It is very simple to understand the method.
3. It can be applied to any type of data, ie quantitative and qualitative
4. It is the only way of studying correlation between qualitative data such as honesty, beauty etc.
5. As the sum of rank differences of the two qualitative data is always equal to zero, this method facilitates a cross check on the calculation.

Demerits of Rank Correlation method

1. Rank correlation coefficient is only an approximate measure as the actual values are not used for calculations.
2. It is not convenient when number of pairs (ie. N) is large
3. Further algebraic treatment is not possible
4. Combined correlation coefficient of different series cannot be obtained as in the case of mean and standard deviation. In case of mean and standard deviation, it is possible to compute combine arithematic mean and combined standard deviation.

Concurrent Deviation Method:

Concurrent deviation method is a very simple method of measuring correlation. Under this method, we consider only the directions of deviations. The magnitudes of the values are completely ignored. Therefore, this method is useful when we are interested in studying correlation between two variables in a casual manner and not interested in degree (or precision).

Under this method, the nature of correlation is known from the direction of deviation in the values of variables. If deviations of 2 variables are concurrent, then they move in the same direction, otherwise in the opposite direction.

The formula for computing the coefficient of concurrent deviation is: - $r = \pm\sqrt{\pm\frac{(2c-N)}{N}}$

Where N = No. of pairs of symbol

C = No. of concurrent deviations (ie, No. of + signs in 'dx dy' column)

Steps:

1. Every value of 'X' series is compared with its proceeding value. Increase is shown by '+' symbol and decrease is shown by '-'
2. The above step is repeated for 'Y' series and we get 'dy'

3. Multiply ‘dx’ by ‘dy’ and the product is shown in the next column. The column heading is ‘dxdy’.
4. Take the total number of ‘+’ signs in ‘dxdy’ column. ‘+’ signs in ‘dxdy’ column denotes the concurrent deviations, and it is indicated by ‘C’.
5. Apply the formula:

$$r = \sqrt[\pm]{\pm\frac{(2c-N)}{N}}$$

If 2c>N, then r = +ve and if 2c < N, then r = -ve .

Qn:- Calculate coefficient if correlation by concurrent deviation method:-

Year :	2003	2004	2005	2006	2007	2008	2009	2010	2011
Supply :	160	164	172	182	166	170	178	192	186
Price :	292	280	260	234	266	254	230	190	200

Sol: Computation of coefficient of concurrent Deviation

Supply (x)	Price (y)	dx	dy	dxdy
160	292	+	-	-
164	280	+	-	-

172	260	+	-	-
182	234	+	-	-
166	266	-	+	-
170	254	+	-	-
178	230	+	-	-
192	190	+	-	-
186	200	-	+	-
				C= 0

$$r = \sqrt[\pm]{\pm\frac{(2c-N)}{N}}$$

$$r = \sqrt[\pm]{\pm\frac{(2\times 0-8)}{8}}$$

$$r = \sqrt[\pm]{\pm\frac{(0-8)}{8}}$$

$$= r = \sqrt[\pm]{\pm\frac{(-8)}{8}} = 1$$

Merits of concurrent deviation method:

1. It is very easy to calculate coefficient of correlation
2. It is very simple understand the method
3. When the number of items is very large, this method may be used to form quick idea about the degree of relationship

4. This method is more suitable, when we want to know the type of correlation (ie, whether positive or negative).

Demerits of concurrent deviation method:

1. This method ignores the magnitude of changes. ie. Equal weight is give for small and big changes.
2. The result obtained by this method is only a rough indicator of the presence or absence of correlation
3. Further algebraic treatment is not possible
4. Combined coefficient of concurrent deviation of different series cannot be found as in the case of arithmetic mean and standard deviation.

Regression Analysis

Correlation analysis analyses whether two variables are correlated or not. After having established the fact that two variables are closely related, we may be interested in estimating the value of one variable, given the value of another. Hence, regression analysis means to analyse the average relationship between two variables and thereby provides a mechanism for estimation or predication or forecasting.

The term ‘Regression” was firstly used by Sir Francis Galton in 1877. The dictionary meaning of the term ‘regression” is “stepping back” to the average.

Definition:

“Regression is the measure of the average relationship between two or more variables in terms of the original units of the date”.

“Regression analysis is an attempt to establish the nature of the relationship between variables-that is to study the functional relationship between the variables and thereby provides a mechanism for prediction or forecasting”.

It is clear from the above definitions that Regression Analysis is a statistical device with the help of which we are able to estimate the unknown values of one variable from known values of another variable. The variable which is used to predict the another variable is called independent variable (explanatory variable) and, the variable we are trying to predict is called dependent variable (explained variable).

The dependent variable is denoted by X and the independent variable is denoted by Y.

The analysis used in regression is called simple linear regression analysis. It is called simple because three is only one predictor (independent variable). It is called linear because, it is assumed that there is linear relationship between independent variable and dependent variable.

Types of Regression:-

There are two types of regression. They are linear regression and multiple regression.

Linear Regression:

It is a type of regression which uses one independent variable to explain and/or predict the dependent variable.

Multiple Regression:

It is a type of regression which uses two or more independent variable to explain and/or predict the dependent variable.

Regression Lines:

Regression line is a graphic technique to show the functional relationship between the two variables X and Y. It is a line which shows the average relationship between two variables X and Y.

If there is perfect positive correlation between 2 variables, then the two regression lines are winding each other and to give one line. There would be two regression lines when there is no perfect correlation between two variables. The nearer the two regression lines to each other, the higher is the degree of correlation and the farther the regression lines from each other, the lesser is the degree of correlation.

Properties of Regression lines:-

1. The two regression lines cut each other at the point of average of X and average of Y (i.eX and Y)
2. When r = 1, the two regression lines coincide each other and give one line.
3. When r = 0, the two regression lines are mutually perpendicular.

Regression Equations (Estimating Equations)

Regression equations are algebraic expressions of the regression lines. Since there are two regression lines, therefore two regression equations. They are :-

Regression Equation of X on Y:-

This is used to describe the variations in the values of X for given changes in Y.

Regression Equation of Y on X :-

This is used to describe the variations in the value of Y for given changes in X.

Least Square Method of computing Regression Equation:

The method of least square is an objective method of determining the best relationship between the two variables constituting a bivariate data. To find out best relationship means to determine the values of the constants involved in the functional relationship between the two variables. This can be done by the principle of least squares:

The principle of least squares says that the sum of the squares of the deviations between the observed values and estimated values should be the least. In other words, Σ will be the minimum.

With a little algebra and differential calculators we can develop some equations (2 equations in case of a linear relationship) called normal equations. By solving these normal equations, we can find out the best values of the constants.

Regression Equation of Y on X:-

$$Y = a + bx$$

The normal equations to compute ‘a’ and ‘b’ are: -

$$\sum y = Na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

Regression Equation of X on Y:-

$$X = a + by$$

The normal equations to compute ‘a’ and ‘b’ are:-

$$\sum x = Na + b\sum y$$

$$\sum xy = a\sum y + b\sum y^2$$

Qn:- Find regression equations x and y and y on x from the following:-

X:	25	30	35	40	45	50	55
Y:	18	24	30	36	42	48	54

Sol: Regression equation x on y is:

$$x = a + by$$

Normal equations are:

$$\sum x = Na + b\sum y$$

$$\sum xy = a\sum y + b\sum y^2$$

Computation of Regression Equations				
x	y	x2	y^2	xy

25	18	625	324	450
30	24	900	576	720
35	30	1225	900	1050
40	36	1600	1296	1440
45	42	2025	1764	1890
50	48	2500	2304	2400
55	54	3025	2916	2970
$\Sigma x = 280$	$\Sigma y = 252$	$\Sigma x^2 = 11900$	$\Sigma y^2 = 10080$	Σxy 10920

$280 = 7a + 252\,b$ ------------ (1)

$10920 = 252a + 10080\,b$ ----------- (2)

Eq. 1×36 $10080 = 252a + 9072b$ ------------- (3)

$\underline{10920 = 252a + 10080b}$ ------------- (2)

(2) × (3) $840 = 0 + 1008\,b$

$1008\,b = 840$

$b = \frac{840}{1008} = 0.83$

Substitute b = 0.83 in equation (1)

$280 = 7a + (252\times0.83)$

$280 = 7\ a + 209.16$

$7a + 209.116 = 280$

$7a = 280-209.160$

$a = \frac{70.84}{7} = 10.12$

Substitute a = 10.12 and b =0.83 in regression equation:

$X = 10.12 + 0.83y$

Regression equation Y on X is:

$y = a + bx$

Normal Equations are:-

$\sum y = Na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$

$252 = 7a + 280\ b$ ------- (1)

$10920 = 280\ a + 11900\ b$ ------- (2)

(1)× 40 → $10080 = 280\ a + 11200\ b$ ------- (3)

$10920 = 280\ a + 11900\ b$ ------- (2)

(2) – (3) → $840 = 0 + 700\ b$

$700\ b = 840$

$b = \frac{840}{700} = 1.2$

Substitute b = 1.2 in equation (1)

$252 = 7a + (280x1.2)$

252 = 7a + 336

7a + 336 = 252

7a = 252 – 336 = -84

$$a = \frac{-84}{7} = -12$$

Substitute a = -12 and b = 1.2 in regression equation

$$y = -12+1.2x$$

Qn:- From the following bivariate data, you are required to: -

(a) Fit the regression line Y on X and predict Y if x = 20

(b) Fir the regression line X on Y and predict X if y = 10

X:	4	12	8	6	4	4	16	8
Y:	14	4	2	2	4	6	4	12

Computation of regression equations

x	y	x^2	y^2	xy
4	14	16	196	56
12	4	144	16	48
8	2	64	4	16
6	2	36	4	12
4	4	16	16	16
4	6	16	36	24

16	4	256	16	64
8	12	64	144	96
Σ x= 62	Σy=48	Σx^2= 612	Σy^2= 432	Σ xy= 332

Regression equation y on x

$$y = a + bx$$

Normal equations are:

$$\sum y = Na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

48 = 8a + 62 b ………………. (1)

332 = 62a + 612 b …………… (2)

eq. 1×62 → 2,976 = 496a+ 3844b...…...(3)

eq. 2×8 → $\underline{2,976 = 496 + 4896b}$ … . . (4)

eq. 3×eq. 4→ 320 = 0+ 1052b

-1052 b = 320

$$b = \frac{320}{-1052}$$

Substitute b = -0.304 in eq (1)

48 = 8a + (62 x -0.304)

48 = 8a + -18.86

48 + 18.86 = 8a

a = 66.86

. $a = \frac{66.86}{8} \qquad = 8.36$

Substitute a = 8.36 and b = -0.304 in regression equation y on x:

$y = 8.36 + -0.3042\ x$

$y = 8.36 – 0.3042\ x$

If x = 20, then,

$y=8.36 – (0.3042x20)$

$= 8.36 – 6.084$

$= \underline{\underline{2.276}}$

(b) Regression equation X on Y:

$X=a + by$

Normal equations:

$\sum x = Na + b\sum y$

$\sum xy =a\sum y + b\sum y^2$

$62 = 8a + 48\ b$ ………….. (1)

$332 = 48\ a + 432\ b$ ……… (2)

eq (1) ×6 → $372=48a+ 288b$…. …. . . (3)

$332 = 48\ a + 432\ b$ ……(2)

Eq(2)-(3) → $-40 = 0+144b$

$144\ b = -40$

$b =\frac{-40}{144} = - 0.2778$

Substitute b = -0.2778 in equation (1)

$$62 = 8a + (48\ 0.\ 2778)$$

$$62 = 8a + -13.3344$$

$$62+13.3344 = 8\ a$$

$$8a = 75.3344$$

$$a = \frac{75.3344}{8} \qquad = 9.4168$$

Substitute a = 9.4168 and b = -0.2778 in regression equation:

$$x = 9.4168 + -0.2778\ y$$

$$x = 9.4168 + -0.2778\ y$$

If y=10, then

$$x=9.4168 – (0.2778x10)$$

$$x= 9.4168 – 2.778$$

$$x = 6.6388$$

Regression Coefficient method of computing Regression Equations:

Regression equations can also be computed by the use of regression coefficients.

Regression coefficient X on Y is denoted as b_{xy} and that of Y on X is denoted as $b_{yx.}$

Regression Equation x on y:

$$\boxed{x - \bar{x} = b_{xy} (y - \bar{y})}$$

ie x -$\bar{x}$ =$r.\frac{\sigma_x}{\sigma_y}$ (y -$\bar{y}$)

Regression Equation y on x:

y -$\bar{y}$ = b_{yx} (x -$\bar{x}$)

ie y -$\bar{y}$ =$r.\frac{\sigma_y}{\sigma_x}$ (x -$\bar{x}$)

Properties of Regression Coefficient:

1. Both regression co-efficient wills have the same sign i.e. either they will be positive or negative. It is never possible that one of the regression co-efficient is negative & other positive.
2. Since the value of the co-efficient of correlation bxy & byx cannot exceed one, one of the regression co-efficient must be less than one or, in other words, both the regression co-efficient cannot be greater than 1.
3. The coefficient of correlation will have the same sign as that of regression co-efficient i.e. if regression co-efficient have a negative sign, r will also be negative and if regression coefficient have a positive sign, r would be positive
4. Correlation coefficient is the geometric mean between regression coefficients

5. Regression is affected by change of scale & independent of change in origin
6. The arithmetic mean of bxy & byx is greater than or equal to coefficient of correlation
7. Since $b_{yx} = r\frac{\sigma_y}{\sigma_x}$ we can find any of these four values, given the other three
8. If σy = σx, then coefficient correlation equal to regression coefficient, r = byx = bxy
9. If r=0 the byx and bxy both are zero
10. If byx = bxy then it is equal coefficient of correlation , r = byx = bxy

Computation of Regression Co-efficients

Regression co-efficients can be calculated in 3 different ways:

1. Actual mean method
2. Assumed mean method
3. Direct method

Actual mean method:-

Regression coefficient x on y (b_{xy}) = $\frac{\Sigma xy}{\Sigma y^2}$

Regression coefficient y on x (b_{yx}) = = $\frac{\Sigma xy}{\Sigma y^2}$

Where x = x –$\bar{x}$

y = y –$\bar{y}$

Assumed mean method:

Regression coefficient x on y (b_{xy}) $\Big\}$ $\frac{\Sigma dxdy-(\Sigma dx).(\Sigma dy)}{\Sigma dy^2-(\Sigma dy)^2}$

Regression coefficient y on x (b_{yx}) $\Big\}$ $\frac{\Sigma dxdy-(\Sigma dx).(\Sigma dy)}{\Sigma dx^2-(\Sigma dx)^2}$

Where dx = deviation from assumed mean of X

dy = deviation from assumed mean of Y

Direct method:-

Regression Coefficient x on y (b_{xy}) $\Big\}$ $\frac{N\Sigma xy-.\Sigma x.\Sigma y)}{N\Sigma y^2-(\Sigma y)^2}$

Regression Coefficient y on x (b_{yx}) $\Big\}$ $\frac{N\Sigma xy-.\Sigma x.\Sigma y)}{N\Sigma x^2-(\Sigma x)^2}$

Qn:- Following information is obtained from the records of a business organization:-

Sales (in '000):

91 53 45 76 89 95 80 65

Advertisement Expense (₹in '000):

15 8 7 12 17 25 20 13

You are required to:-

Compute regression coefficients under 3 methods

Obtain the two regression equations and

Estimate the advertisement expenditure for a sale of Rs. 1,20,000

Let x = sales y = Advertisement expenditure

Computation of regression Coefficients under actual mean method						
x	y	**x -**$\bar{x}$	**y -**$\bar{y}$	xy	x^2	y^2
91	15	16.75	0.375	6.28	280.56	0.14
53	8	-21.65	-6.625	140.78	451.56	43.89
45	7	-29.25	-7.625	223.03	855.56	58.14
76	12	1.75	-2,625	-4.59	3.06	6.89
89	17	14.75	-2.375	35.03	217.56	5.64
95	25	20.75	10.375	215.28	430.56	107.64
80	20	5.75	5.375	30.91	33.06	28.89
65	13	-9.25	-1.625	15.03	85.56	2.64
Σ x= 594	Σy= 117			Σxy=661.75	Σx^2= 2357.48	Σy^2=253.87

$$\overline{X} = \frac{\Sigma x}{N} = \frac{294}{8} = 74.25$$

$$\bar{Y} = \frac{\sum y}{N} = \frac{117}{8} = 14.625$$

Regression Coefficient x on y

$$(b_{xy})\ \frac{\sum xy}{\sum y^2} = \Big\}\ \frac{661.75}{253.87} = 2.61$$

Regression Coefficient y on x $b_{yx})$

$$\Big\}\ \frac{\sum xy}{\sum x^2} = \frac{661.75}{2357.48} = 0.28$$

Computation of Regression Coefficient under assumed mean method

x	y	x-70 (dx)	y-15 (dy)	dxdy	dx^2	dy^2
91	15	21	0	0	441	0
53	8	-17	-7	+119	289	49
45	7	-25	-8	+200	625	64
76	12	6	-3	-18	36	9
89	17	19	2	+38	361	4
95	25	25	10	+250	625	100
80	20	10	5	+50	100	25
65	13	-5	-2	+10	25	4
		dx= 34	Σdy= -3	Σdxdy =	Σdx^2=	Σdy^2= 255

649 2502

Regression coefficient x on y (b_{xy}) = $\frac{\Sigma dxdy-(\Sigma dx).(\Sigma dy)}{\Sigma dy^2-(\Sigma dy)^2}$

$= \frac{(8\times649)-(34\times-3)}{(8\times255)-(-3)^2}$

$= \frac{5192--102}{2040-9}$

$= \frac{5294}{2031}$

= 2.61

Regression coefficient y on x (b_{yx}) = $\frac{\Sigma dxdy-(\Sigma dx).(\Sigma dy)}{\Sigma dx^2-(\Sigma dx)^2}$

$= \frac{(8\times649)-(34\times-3)}{(8\times2502)-(34)^2}$

$= \frac{5192--102}{20016-1156}$

$= \frac{5294}{18860}$

= 0.28

Computation of Regression Coefficient under direct method				
x	y	xy	x^2	y^2
91	15	1365	8281	225
53	8	424	2809	64
45	7	315	2025	49
76	12	912	5776	144

89	17	1513	7921	289
95	25	2375	9025	625
80	20	1600	6400	400
65	13	845	4225	169
Σx= 594	Σy= 117	Σxy= 9349594	Σx^2 = 46462	Σy^2 = 1965

Regression Coefficient x on y (b_{xy}) $= \frac{\text{N}\Sigma xy - .\Sigma x.\Sigma y)}{\text{N}\Sigma y^2 - (\Sigma y)^2}$

$$= \frac{(8\times9349)-(594\times117)}{(8\times1965)-(117)^2}$$

$$= \frac{74792-69498}{15720-13689}$$

$$= \frac{5294}{2031}$$

$$= 2.61$$

Regression Coefficient y on x (b_{yx}) $= \frac{\text{N}\Sigma xy - .\Sigma x.\Sigma y)}{\text{N}\Sigma x^2 - (\Sigma x)^2}$

$$= \frac{(8\times9349)-(594\times117)}{(8\times46462)-(594)^2}$$

$$= \frac{74792-69498}{371696-352836}$$

$$= \frac{5294}{18860}$$

$$= 0.28$$

3) a) Regression equation X on Y:

$(x-\bar{x}) = b_{xy}(y-\bar{y})$

$(x-74.25) = 2.61(\bar{y}-14.625)$

$(x-74.25) = 2.61y-38.17$

$x = 74.25 – 38.17+2.61y$

$x = \underline{\underline{36.08 + 2.61y}}$

b) Regression equation y pm x:

$(y-\bar{y}) = b_{yx}(x-\bar{x})$

$(y-14.625) = 0.28(x-74.25)$

$y =14.625 = 0.28(x-20.79)$

$y = 14.625 – 20.79 + 0.28x$

$y = -6.165+0.28x$

$y = 0.28x - 6.165$

4)If sales (x)isRs. 1,20,000, then

Estimated advertisement Exp (y) = (0.28x120)-6.165

= (33.6 – 6.165)

= 27.435

i.e = Rs. $\underline{\underline{27,435}}$

Qn: In a correlation study, the following values are obtained:

Mean $\frac{x}{65}$ $\frac{y}{67}$

Standard deviation 2.5 3.5

Coefficient of correlation 0.8

Find the regression equations

Sol: Regression equation x on y is:

$$x- \bar{x} = b_{xy} (y- \bar{y})$$

$$x- \bar{x} = r.\frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$x - 65 = 0.8\frac{2.5}{3.5} (y-67)$$

$$x- 65 = 0.5714 (y-67)$$

$$x - 65 = 0.5714y-38.2838$$

$$x = 65 – 38.2838+0.5714y$$

$$x = 26.72 + 0.5714y$$

Regression equation y on x is:

$$y- \bar{y} = b_{xy} (x- \bar{x})$$

$$y- \bar{y}= r.\frac{\sigma_x}{\sigma_y} (x- \bar{x})$$

$$x - 67 = 0.8\frac{3.5}{2.5} (x-65)$$

$$y - 67 = 1.12 (x-65)$$

$$y = 67 – (1.12 x65) = 1.12 x$$

$$y = 67.72.8 + 1.12x$$

$$y = -5.8 +1.12x$$

$$y = 1.12x - 5.8$$

Qn: Two variables gave the following data

$\bar{x}$ = 20, σ_x = 4, r = 0.7

$\bar{y}$ = 15, σ_x = 3

Obtain regression lines and find the most likely value of y when x=24

Sol: Regression Equation x on y is

$$x- \bar{x} = b_{xy} (y- \bar{y})$$

$$x- \bar{x} = r.\frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$x - 20 = 0.7\frac{4}{3} (y-15)$$

$$x - 20 = \frac{2.8}{3}x (y-15)$$

$$x - 20 = 0.93 (y-15)$$

$$x = 20 + 0.93y – 13.95$$

$$x = 20 - 13.95 + 0.93y$$

$$x = 6.05 + 0.93y$$

Regression Equation y on x is

$$y- \bar{y} = b_{xy} (x- \bar{x})$$

$$y- \bar{y}= r.\frac{\sigma_x}{\sigma_y} (x- \bar{x})$$

$$y -\bar{y}= 0.7\frac{3}{4} (x-20)$$

$$y -15 = 0.525(x – 20)$$

$$y – 15 = 0.525 x -10.5$$

$y = 15-10.5+ 0.525x$

$y =4.5+0.525x$

If $X = 24$,

then $y = 4.5 + (0.525\times24)$

$y = 4.5 + 12.6$

$y = 17.1$

Qn: For a given set of bivariate data, the following results were obtained:

$\bar{x} = 53.2$, $\bar{y} = 27.9$, $b_{yx} = -1.5$ and $b_{xy} = -0.2$

Find the most probable value of y when x = 60. Also find ‘r’.

Sol: Regression Equation y on x is:

$y- \bar{y} = b_{xy} (x- \bar{x})$

$y - 27.9 = -1.5 (x-53.2)$

$y - 27.9 =-1.5x + 79.8$

$y = 79.8+27.9 - 1.5x$

$y = 107.7 - 1.5x$

If $x = 60$, then

$y = 107.7 - (1.5\times 60)$

$= 107.7-90$

$= 17.7$

$r = \sqrt{b_{xy} \times b_{yx}}$

$= -\sqrt{1.5\ \times 0.2} = -\sqrt{}30$

$= -0.5477$

Correlation		Regression
1	It studies degree of relationship between variables	It studies the nature of relationship between variables
2	It is not used for prediction purposes	It is basically used for prediction purposes
3	It is basically used as a tool for determining the degree of relationship	It is basically used as a tool for studying cause and effect relationship
4	There may be nonsense correlation between two variables	There is no such nonsense regression
5	There is no question of dependent and independent variables	There must be dependent and independent variables

Module IV: Probability

Probability refers to the chance of happening or not happening of an event. In our day today conversations, we may make statements like "probably he may get the selection", "possibly the Chief Minister may attend the function", etc. Both the statements contain an element of uncertainly about the happening of the even. Any problem which contains uncertainty about the happening of the event is the problem of probability.

Definition of Probability

The probability of given event may be defined as the numerical value given to the likely hood of the occurrence of that event. It is a number lying between '0' and '1' '0' denotes the even which cannot occur, and '1' denotes the event which is certain to occur. For example, when we toss on a coin, we can enumerate all the possible outcomes (head and tail), but we cannot say which one will happen. Hence, the probability of getting a head is neither 0 nor 1 but between 0 and 1. It is 50% or ½

Terms use in Probability.

Random Experiment

A random experiment is an experiment that has two or more outcomes which vary in an unpredictable manner from trial to trail when conducted under uniform conditions.

In a random experiment, all the possible outcomes are known in advance but none of the outcomes can be predicted with certainty. For example, tossing of a coin is a random experiment because it has two outcomes (head and tail), but we cannot predict any of them which certainty.

Sample Point

Every indecomposable outcome of a random experiment is called a sample point. It is also called simple event or elementary outcome.

Eg. When a die is thrown, getting ‘3’ is a sample point.

Sample space

Sample space of a random experiment is the set containing all the sample points of that random experiment.

Eg:- When a coin is tossed, the sample space is (Head, Tail)

Event

An event is the result of a random experiment. It is a subset of the sample space of a random experiment.

Sure Event (Certain Event)

An event whose occurrence is inevitable is called sure even.

Eg:- Getting a white ball from a box containing all while balls.

Impossible Events

An event whose occurrence is impossible, is called impossible event. Eg:- Getting a white ball from a box containing all red balls.

Uncertain Events

An event whose occurrence is neither sure nor impossible is called uncertain event.

Eg:- Getting a white ball from a box containing white balls and black balls.

Equally likely Events

Two events are said to be equally likely if anyone of them cannot be expected to occur in preference to other. For example, getting herd and getting tail when a coin is tossed are equally likely events.

Mutually exclusive events

A set of events are said to be mutually exclusive of the occurrence of one of them excludes the possibiligy of the occurrence of the others.

Exhaustive Events:

A group of events is said to be exhaustive when it includes all possible outcomes of the random experiment under consideration.

Dependent Events:

Two or more events are said to be dependent if the happening of one of them affects the happening of the other.

Different Schools of Thought on Probability

There are 4 important schools of thought on probability :-

1. Classical or Priori Approach
2. Relative frequency or Empirical Approach
3. Subjective or Personalistic Approach
4. Modern or Axiomatic Approach

1. Classical or Priori Approach

If out of '*n*' exhaustive, mutually exclusive and equally likely outcomes of an experiment; '*m*' are favourable to the occurrence of an event 'A', then the probability of 'A' is defined as to be $\frac{m}{n}$

$$P(A) = \frac{m}{n}$$

According to Laplace, a French Mathematician, "the probability is the ratios of the number of favourable cases to the total number of equally likely cases."

$$P(A) = \frac{number\ of\ favourable\ cases}{total\ number\ of\ equally\ likely\ cases}$$

Question

What is the chance of getting a head when a coin is tossed?

Total number of cases = 2

No. of favorable cases = 1

Probability of getting head = $\frac{1}{2}$

Question

A die is thrown. Find the probability of getting.

a ‘4’

an even number

‘3’ or ‘5’

less than ‘3’

Solution

Sample space is (1,2, 3, 4, 5, 6)

Probability (getting ‘4) = $\frac{1}{6}$

Probability (getting an even number) = $\frac{3}{6} = \frac{1}{6}$

Probability (getting 3 or 5) = $\frac{2}{6} = \frac{1}{3}$

Probability (getting less than ‘3’) = $\frac{2}{6} = \frac{1}{3}$

Question

A ball is drawn from a bag containing 4 white, 6 black and 5 yellow balls. Find the probability that a ball drawn is:-

(1) White (2) Yellow (3) Black (4) Not yellow

(5) Yellow or white

Solution

P (drawing a white ball) = $\frac{4}{15}$

P (drawing a yellow ball) = $\frac{5}{15}$ = $\frac{1}{3}$

P (drawing a black ball) = $\frac{6}{15}$ = $\frac{2}{5}$

P (drawing not a yellow ball) = $\frac{10}{15}$ = $\frac{2}{3}$

P (drawing a yellow or white ball) = $\frac{9}{15}$ = $\frac{3}{5}$

Question

There are 19 cards numbered 1 to 19 in a box. If a person drawn one at random, what is the probability that the number printed on the card be an even number greater than 10?

Solution

The even numbers greater than 10 are 12, 14, 16 and 18.

∴P (drawing a card with an even number greater than 10). = $\frac{4}{9}$

Question

Two unbiased dice are thrown. Find the probability that :-

Both the dice show the same number

One die shows 6

First die shows 3

Total of the numbers on the dice is 9

Total of the numbers on the dice is greater than 8

A sum of 11

Solution

When 2 dice are thrown the sample space consists of the following outcomes :-

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,3)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

P(that both the dice shows the same number) = $\frac{6}{36}$= $\frac{1}{6}$

P (that one die shows 6) =$\frac{10}{36}$= $\frac{5}{18}$

P (that first die shows 3) = $\frac{6}{36}$ = $\frac{1}{6}$

P (that total of the numbers on the dice is 9) =$\frac{4}{36}$ =$\frac{1}{9}$

P (that total of the number is greater than 8) = $\frac{10}{36}$= $\frac{5}{18}$

P (that a sum of 11) = $\frac{2}{36}$= $\frac{1}{18}$

Limitations of Classical Definition:

1. Classical definition has only limited application in coin-tossing die throwing etc. It fails to answer question like

"What is the probability that a female will die before the age of 64?"

2. Classical definition cannot be applied when the possible outcomes are not equally likely. How can we apply classical definition to find the probability of rains? Here, two possibilities are "rain" or "no rain". But at any given time these two possibilities are not equally likely.
3. Classical definition does not consider the outcomes of actual experimentations.

2. Relative Frequency Definition or Empirical Approach

According to Relative Frequency definition, the probability of an event can be defined as the relative frequency with which it occurs in an indefinitely large number of trials.

If an even 'A' occurs 'f' number of trials when a random experiment is repeated for 'n' number of times, then

$$P(A) = \text{Lt}\ \underset{n \to \propto}{Lt} \frac{f}{n}$$

For practical convenience, the above equation may be written as $P(A) = \frac{f}{n}$

Here, probability has between 0 and 1,

i.e. $0 \leq P(A) \leq 1$

Question

The compensation received by 1000 workers in a factory are given in the following table :-

Wages:	80-100	100-120	120-140	140-160	160-180	180-200
No. of workers	10	100	400	250	200	40

Find the probability that a worker selected has

(1) Wages under Rs.100/-

(2) Wages above Rs.140/-

(3) Wages between Rs. 120/- and Rs.180/-

Solution

P(that a worker selected has wages under Rs.140/-)

$$= \frac{10+100+400}{1000} = \frac{510}{1000}$$

P(that a worker selected has wages above Rs.140/- =

$$= \frac{250+200+40}{1000} = \frac{490}{1000}$$

P(that a worker selected has wages between 120 and 180)

$$= \frac{400+250+200}{1000} = \frac{850}{1000}$$

3. Subjective (Personalistie) Approach to Probability

The exponents of personalistie approach define probability as a measure of personal confidence or belief based on whatever evidence is available. For example, if a teacher wants to find out the probability that Mr. X topping in M.Com examination, he may assign a value between zero and one according to his degree of belief for possible occurrence. He may take into account such factors as the past academic performance in terminal examinations etc. and arrive at a probability figure. The probability figure arrived under this method may vary from person to person. Hence it is called subjective method of probability.

4. Axiomatic Approach (Modern Approach) to Probability

Let ‘S’ be the sample space of a random experiment, and ‘A’ be an event of the random experiment, so that ‘A’ is the subset of ‘S’. Then we can associate a real number to the event ‘A’. This number will be called probability of ‘A’ if it satisfies the following three axioms or postulates:-

1. The probability of an event ranges from 0 and 1.

   If the event is certain, its probability shall be 1.

   If the event cannot take place, its probability shall be zero.

2. The sum of probabilities of all sample points of the sample spece is equal to 1. i.e, $P(S) = 1$

3. If A and B are mutually exclusive (disjoint) events, then the probability of occurrence of either A or B shall be :

   $P(A\ B) = P(A) + P(B)$

Theorems of Probability

There are two important theorems of probability. They are :

i. Addition Theorem

ii. Multiplication Theorem

Addition Theorem

Here, there are 2 situations.

a. Events are mutually exclusive

b. Events are not mutually exclusive

(a) Addition theorem (Mutually Exclusive Events)

If two events, ‘A’ and ‘B’, are mutually exclusive the probability of the occurrence of either ‘A’ or ‘B’ is the sum of the individual probability of A and B.

P(A or B) = P(A) + P(B)

i.e., P(A B) = P(A) + P(B)

(b)Addition theorem (Not mutually exclusive events)

If two events, A and B are not mutually exclusive the probability of the occurrence of either A or B is the sum of their individual probability minus probability for both to happen.

P(A or B) = P(A) + P(B) – P(A and B)

i.e., P(A B) = P(A) + P(B) – P(A∩B)

Question

What is the probability of picking a card that was red or black?

Solution

Here the events are mutually exclusive

P(picking a red card) = $\frac{26}{52}$

P(picking a black card) = $\frac{26}{52}$

P (picking a red or black card) =$\frac{26}{52}+\frac{26}{52}$ = 1

Question

The probability that a contractor will get a plumping contract is $\frac{2}{3}$ and the probability that he will not get an electric contract is $\frac{5}{9}$. If the probability of getting at least one contract is $\frac{4}{5}$, what is the probability that he will get both the contracts?

Solution

P(getting plumbing contract) = $\frac{2}{3}$

P(not getting electric contract) = $\frac{5}{9}$

P(getting electric contract) = 1 - $\frac{5}{9} = \frac{4}{5}$ =

P(getting at least one contract) = P (getting electric contract) + P(getting plumbing contract) - P(getting both)

i.e., $\frac{4}{5}$= $\frac{4}{9}+\frac{2}{3}-$P(getting both)

P(getting both contracts) = $\frac{4}{9}+\frac{2}{3}$ - $\frac{4}{5}$

$$= \frac{20+30-36}{45} = \frac{14}{45}$$

Question

If P (A) = 0.5, P(B) = 0.6, P(A∩B) = 0.2, find:-

P(A∪ B)

P(A')

P(A ∩ B')

P(A'∩ B')

Solution

Here the events are not mutually exclusive:-

P(A∪ B) = P(A) + P(B) – P(A ∩ B)

= 0.5 + 0.6 – 0.2

= 0.9

P(A') = 1- P(A)

= 1 – 0.5

=0.5

P(A∩ B')

= P(A) – P(A ∩B)

= 0.5 – 0.2

= 0.3

P(A'∩ B') = 1 – P(A∪ B)

= 1- [P(A) + P(B) – P(A∩ B)

= 1 – (0.5 + 0.6 – 0.2)

= 1- 0.9

= 0.1

Multiplication Theorem

Here there are two situations:

Events are independent

Events are dependent

(a)Multiplication theorem (independent events)

If two events are independent, then the probability of occurring both will be the product of the individual probability

P(A and B) = P(A).P(B)

i.e., P(A∩B) = P(A).P(B)

Question

A bag contains 5 white balls and 8 black balls. One ball is drawn at random from the bag and is then replaced. Again another one is drawn. Find the probability that both the balls are white.

Solution

Here the events are independent

P (drawing white ball in I draw) = $\frac{5}{13}$

P (drawing white ball in II draw) =$\frac{5}{13}$

P(drawing white ball in both draw) = $\frac{5}{13} \times \frac{5}{13}$

$$= \frac{25}{169}$$

Question

Single coin is tossed for three tones. What is the probability of getting head in all the 3 times?

Solution

P (getting head in all the 3 times) = P (getting H in 1st toss) × P (getting Head in 2nd toss)× P (getting H in 3rd toss)

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

(b)Multiplication theorem (dependent Events):-

If two events, A and B are dependent, the probability of occurring 2nd event will be affected by the outcome of the first.

$P(A \cap B) = P(A).P(B/A)$

Question

A bag contains 5 white balls and 8 black balls. One ball is drawn at random from the bag. Again, another one is drawn without replacing the first ball. Find the probability that both the balls drawn are white.

Solution

P (drawing a white ball in Ist draw) = $\frac{5}{13}$

P (drawing a white ball in IInd draw) $=\frac{4}{12}$

$= \frac{20}{156}$

Question

The probability that 'A' solves a problem in Maths is $\frac{2}{5}$ and the probability that 'B' solves it is $\frac{3}{8}$. If they try independently find the probability that:-

Both solve the problem.

at least one solve the problem.

none solve the problem.

Solution

P (that both solve the problem) = P(that A solves the problem)×

P (that B solves the problem)

$= \frac{2}{5} \times \frac{3}{8} = \frac{6}{40} = \frac{3}{20}$

P(that at least one solve the problem) = P(that A or B solves the problem)

= P(A solve the problem + P(B solve the problem – P(A and B solve the problem)

$= \frac{2}{5} + \frac{3}{8} - (\frac{2}{5} \times \frac{3}{8})$

$$=\frac{2}{5}+\frac{3}{8}-\frac{6}{40}$$

$$=\frac{16+15-6}{40}$$

$$=\frac{25}{40}=\frac{5}{8}$$

P(that none solve the problem)

=1-P(at least one solve the problem)

= 1 – P (A or B solve the problem)

=1- [P(A solve the problem + P(B solve the problem) - P(A & B solve the problem)]

$$= \mathbf{1}-\left[\frac{\mathbf{2}}{\mathbf{5}}+\frac{\mathbf{3}}{\mathbf{8}}-\left(\frac{\mathbf{2}}{\mathbf{5}}\times\frac{\mathbf{3}}{\mathbf{8}}\right)\right]$$

$$= \mathbf{1}-\left[\frac{\mathbf{2}}{\mathbf{5}}+\frac{\mathbf{3}}{\mathbf{8}}-\frac{\mathbf{6}}{\mathbf{40}}\right]$$

$$= \mathbf{1}-\left[\frac{\mathbf{16+15-6}}{\mathbf{40}}\right]$$

$$= \mathbf{1}-\frac{\mathbf{25}}{\mathbf{40}}=\frac{\mathbf{15}}{\mathbf{40}}=\frac{\mathbf{3}}{\mathbf{8}}$$

Question

A university has to select an examiner from a list of 50 persons. 20 of them are women and 30 men. 10 of them know Hindi and 40 do not. 15 of them are teachers and remaining are not. What is the probability that the university selecting a Hindi knowing woman teacher?

Solution

Here the events are independent.

P(selecting Hindi knowing Woman teacher)= P(selecting Hindi knowing person, woman and teacher)

P(selecting Hindi Knowing persons) = $\frac{10}{50}$

P(selecting woman) = $\frac{20}{50}$

P (selecting teacher) = $\frac{15}{50}$

P(selection Hindi knowing Woman teacher)

$$= \frac{10}{50} \times \frac{20}{50} \times \frac{15}{50}$$

$$= \frac{2}{10} \times \frac{4}{10} \times \frac{3}{10} = \frac{24}{1000}$$

$$= \frac{3}{125}$$

Question

‘A’ speaks truth in 70%cases and ‘B’ in 85% cases. In what percentage of cases they likely to contradict each other in stating the same fact?

Let P(A) = Probability that ‘A’ speaks truth.

P(A’) = Probability that ‘A’ does not speak truth

P(B) = Probability that ‘B’ speaks truth

P(B’) = Probability that ‘B’ does not speak truth

P(A) = 70% = 0.7

P(A’) =30% =0.3

P(B) = 85% = 0.85

P(B’) = 15% = 0.15

∴ P (A and B contradict each other) = P(‘A’ speaks truth and ‘B’ does not OR, A does not speak truth & B speaks)

=P(A & B’) ∪(A’ & B)

=(0.7 ×0.15) + (0.3 ×0.85)

=0.105 + 0.255

= 0.360

Percentage of cases in which A and B contradict each other

= 0.360 ×100

= $\underline{\underline{36\ \%}}$

Question

20% of students in a university are graduates and 80% are undergraduates. The probability that graduate student is married is 0.50 and the probability that an undergraduate student is married is

0.10. If one student is selected at random, what is the probability that the student selected is married?

P(selecting a married student) =

=P (Selecting a graduate married student or selecting an undergraduate married student)

= P (Selecting a graduate & married OR selecting un undergraduate & married)

= (20% ×0.50) + (80% ×0.10)

$= (\frac{20}{100} \times 0.50) + (\frac{80}{100} \times 0.10)$

= (0.2 × 0.50) + (0.8 × 0.1)

= 0.1 +0.08

= 0.18

Question

Two sets of candidates are competing for the position on the board of directors of a company. The probability that the first and second sets will win are 0.6 and 0.4 respectively. If the first set wins, the probability of introducing a new product is 0.8 and the corresponding probability if the second set wins is 0.3. What is the probability that the new product will be introduced?

Solution

P(that new product will be introduced) = P(that new product is introduced by first set OR the

= new product is introduced by second set)

= P (Ist set wins & Ist introduced the new produced OR IInd set wins the new product)

= (0.6 × 0.8) + (0.4 × 0.3)

= 0.48 + 0.12

= 0.60

Question

A certain player say Mr. X is known to win with possibility 0.3 if the truck is fast and 0.4 if the track is slow. For Monday there is a 0.7 probability of a fast track and 0.3 probability of a slow track. What is the probability that Mr. X will win on Monday? **Solution**

P(X will won on Monday) = P(to win in fast track OR to win in slow track)

= P (to get fast track & to win OR to get slow track & to win)

= (0.7 × 0.3) + (0. 3 × 0. 4)

= 0.21 + 0.12

= 0.33

Conditional Probability

Multiplication theorem states that if two events, A and B, are dependent events then, the probability of happening both will be the product of P(A) and P(B/A).

i.e., P(A and B) or P(A∩ B) = P(A).P(B/A)

Here, P (B/A) is called Conditional probability

$$P\ (A\cap B) = P(A).P\ ({}^{B}/_{A})$$

$$\text{i.e., } P(A).P({}^{B}/_{A}) = P(A\cap B)$$

$$\therefore P(\,{}^{B}/_{A}) = \frac{P\ (A\cap B)}{P(A)}$$

$$\text{Similarly, } P\ ({}^{A}/_{B}) \quad = \frac{P\ (A\cap B)}{P(B)}$$

If 3 event, A, B and C and dependent events, then the probability of happening A, B and C is :-

$$P(A\cap B\cap C) \qquad = P(A).\ P\ ({}^{B}/_{A}).\ P\ ({}^{C}/_{AB})$$

$$\text{i.e., } P(A).\ P\ ({}^{B}/_{A}).\ P\ ({}^{C}/_{AB}) = P\ (A\cap B\cap C)$$

$$P\ ({}^{C}/_{AB})0 \qquad = \frac{P(A\cap B\cap C)}{P(A).P\ ({}^{B}/_{A})}$$

Question

If P(A) =$\frac{1}{13}$, P(B)= $\frac{1}{4}$ and P(A∩B) = $\frac{1}{52}$, find:-

P(A/B)

P(B/A)

Solution

Here we know the events are dependent

$$\text{(a) P (A/B) = } \frac{P\ (A\cap B)}{P(B)} = \frac{\frac{1}{52}}{\frac{1}{4}} = \frac{1}{52} \times \frac{4}{1} = \frac{4}{52} = \frac{1}{13}$$

$$\text{(b) P (B/A) = } \frac{P\ (A\cap B)}{P(A)} = \frac{\frac{1}{52}}{\frac{1}{13}} = \frac{1}{52} \times \frac{13}{1} = \frac{13}{52} = \frac{1}{4}$$

Inverse Probability

If an event has happened as a result of several causes, then we may be interested to find out the probability of a particular cause of happening that events. This type of problem is called inverse probability.

Baye’s theorem is based upon inverse probability.

Baye’s Theorem:

Baye’s theorem is based on the proposition that probabilities should revised on the basis of all the available information. The revision of probabilities based on available information will help to reduce the risk involved in decision-making. The probabilities before revision is called priori probabilities and the probabilities after revision are called posterior probabilities.

According to Baye’s theorem, the posterior probability of event (A) for a particular result of an investigation (B) may be found from the following formula:-

$$P(A/B) = \frac{P\ (A).P\ (B)}{P\ (A).P\ (B) + P\ (Not\ A).P\frac{B}{Not}A}$$

Steps in computation

1. Find the prior probability
2. Find the conditional probability.
3. Find the joint probability by multiplying step 1 and step 2.
4. Find posterior probability as percentage of total joint probability.

Question

A manufacturing firm produces units of products in 4 plants, A, B, C and D. From the past records of the proportions of defectives produced at each plant, the following conditional probabilities are set:-

A: 0.5; B: 0.10; C:0.15 and D:0.02

The first plant produces 30% of the units of the output, the second plant produces 25%, third 40% and the forth 5%

A unit of the products made at one of these plants is tested and is found to be defective. What is the probability that the unit was produced in Plant C.

Solution

Computation of Posterior probabilities				
Machine	Priori Probability	Conditional Probabilities	Joint Probability	Posterior Probability
A	0.30	0.05	0.015	$\frac{0.015}{0.101} = 0.1485$
B	0.25	0.10	0.025	$\frac{0.025}{0.101} = 0.2475$
C	0.40	0.15	0.060	$\frac{0.060}{0.101} = 0.5941$
D	0.05	0.02	0.001	$\frac{0.001}{0.101} = 0.0099$
			0.101 ====	1.0000 =====

Probability that defective unit was produced in Machine C = 0.5941

Question

In a bolt manufacturing company machine I, II and III manufacture respectively 25%, 35% and 40%. Of the total of their output, 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the products and is found to be defective. What are the probability that it was manufactured by :-

(a)Machine I

(b) Machine II

(c)Machine III

Solution

Computation of Posterior probabilities				
Machine	Priori Probability	Conditional Probabilities	Joint Probability	Posterior Probability

I	0.25	0.05	0.0125	0.362
II	0.35	0.04	0.0140	0.406
III	0.40	0.02	0.0080	0.232
			0.0345 ====	1.000 =====

P(that the bolt was manufactured by Machine I) = 0.362

P(that the bolt was manufactured by Machine II) = 0.406

P(that the bolt was manufactured by Machine III) = 0.232

Question

The probability that a doctor will diagnose a particular disease correctly is 0.6. The probability that a patient will die by his treatment after correct diagnosis is 0.4 and the probability of death by wrong diagnosis is 0.7. A patient of the doctor who had the disease died. What is the probability that his disease was not correctly diagnosed?

Solution

Computation of Posterior probabilities				
Nature of Diagnosis	Priori Probability	Conditional Probabilities	Joint Probability	Posterior Probability
Correct	0.6	0.4	0.24	$\frac{024}{0.52} = 0.462$
Not correct	0.4	0.7	0.28	$\frac{028}{0.52} = 0.538$
			0.52	1. 000

Probability that the disease was not correctly diagnosed = 0.538

Question

There are two Urns, one containing 5 white balls and 4 black balls; and the other containing 6 white balls and 5 black balls. One Urn is chosen and one ball is drawn. If it is white, what is the probability that the Urn selected is the first?

Solution

Computation of Posterior probabilities				
No.of Urn	Priori Probability	Conditional Probabilities	Joint Probability	Posterior Probability
Correct	$\frac{5}{9}$	$\frac{1}{2}$	$\frac{5}{18} = 0.2778$	$\frac{0.2778}{0.5505}$ $= 0.5046$
Not correct	$\frac{6}{11}$	$\frac{1}{2}$	$\frac{6}{22} = 0.2727$	$\frac{0.2727}{0.5505}$ $= 0.4954$
			. . 0.5505	1. 000

P(that the white balls drawn is from Urnn I = 0.5046

Set theory

The theory of sets was introduced by the German mathematician Georg Cantor in 1870. A set is well defined collection of distinct objects. The term well defined we mean that there exists a rule with the help of which we will be able to say whether a particular object 'belong to' the set or does not belong to the set. The objects in a set are called its members or elements.

The sets are usually denoted by the Capital letters of the English alphabet and the elements are denoted by small letters.

If x is an element of a set A, we write X∈A (read as x belongs to A). If x is not an element of A then we write X∉A (read as x does not belong to A).

Representation of a Set or Methods of describing a Set

A set is often representation in two ways:

a. Roster method or tabular or enumeration method.

b. Set builder method or Rule or Selector method.

Tabular Method

In this method, a set is described by listing the elements, separated by commas and are enclosed within braces. For example the set of first three odd numbers 1,3,5 is represented as :

A = {1, 3, 5}

Set Builder Method

In this method, the set is represented by specifying the characteristic property of its elements. For example the set of natural numbers between 1 and 25 is represented as:

A = {x: x ∈ N and $1 < x < 25$}

Types of Sets

1. Null Set or Empty Set or Void Set

   A set containing no element is called a null set. It is denoted by { } or Ø

   Eg:- the set of natural numbers between 4 and 5.

2. Singleton or Unit Set

   A Set containing a single element is called singleton set

   Eg:- Set of all positive integers less than 2

3. Finite Set

   A Set is said to be a finite set if it consist only a finite number of elements. The null setis regarded as a finite set.

   Eg:- the set of natural numbers less than 10

4. Infinite Set

   A set is said to be an infinite set if it consists of a infinite number of elements.

Eg:- Set of natural numbers.

5. Equvilant Set

Two sets A and B are said to be equivalent set if they contain the same number ofelements

Eg:- Let A = {1, 2, 3 } and B = {a, b, c }

6. Equal Set

Two sets A and B are said to be equal if they contain the same elements.

Eg:- Let A = {1, 2, 3 } B = {2, 1, 3 }

7. Sub Set and Super Set

If every element of A is an element of B then A is called a subset of B and symbolicallywe write A⊆B

If A is contains in B then B is called super set of A and written as B ⊇A

Eg: A = {2, 3} and B = {2, 3, 4} then A is a proper subset of B

8 . Power Set

The collection of all sub sets of a set A is called the power set of A. It is denoted by P (A). In P (A), every element is a set.

For example A = {1, 2, 3}

Then P(A) ={ }, {1, 2, 3 } {1}{ 2, } {3 }{1, 2 } {1, 3 } {2, 3 }

9. Universal Set

If all the sets under consideration are subsets of a fixed set U, is called universal set.

For example A is the set of vowels in the English Alphabet. Then the set of all letters of the English Alphabet may be taken as the universal set.

10. Disjoint Set

Two sets A and B are said to be disjoint sets if no element of A is in B and no element of B is in A

For example A= {3, 4, 5 }, B = {6, 7, 8 }

Set Operations

1. Union of sets :

   The union of two sets A and B is the set of all those elements which belongs to A or to B or to both. We use the notation AUB to denote the union of A and B.

   For example If A = {1, 2, 3, 4 } B = {3, 4, 5, 6 } , Then AUB = {1, 2, 3, 4, 5, 6 }

2. Intersection of Sets

   The intersection of two sets is the set consisting of all elements which belong to both A and B. It is denoted by A∩B.

   For example:If A = {1, 2, 3, 4 } B = {3, 4, 5, 6 } , Then

A∩B = {3, 4 }

3 . Difference of two sets

The difference of the two sets A and B is the set of all elements in A which are not in B. It is denoted by A-B or A/B.

For example: If A = {1, 2, 3, 4 } B = {3, 4, 5, 6 } , Then A—B = {1, 2 }

4. Complement of a set

Complement of a set is the set of all elements belonging to the universal set but not belonging to A. It is denoted by A^c or A'

A^c= U-A.

For example: If U= {1, 2, 3, 4,5 } A ={1,3, 5} ,Then A^c = { 2, 4 }

Algebra of Sets or Laws of Set Operation

1. Commutative Laws :-

If A and B are any two sets then :-

(i)AUB =BUA

(ii)A∩B = B∩A

2. Associative Laws

If A, B and C are three sets, then

(i)AU (BUC) = (AUB) UC and

(ii)A∩ (B∩C) = (A∩B) ∩C

3. Distributive Laws

If A, B, C are any three sets, then

(i) AU (B∩C) = (AUB) ∩(AUC) and

(ii) A∩ (BUC) = (A∩B) U (A ∩C)

4.De-Morgan's Law

If A and B are any two subsets of 'U', then

(i) (AUB)' =A'∩B'

That is complement of union of two sets equal to the intersection of their complements.

(ii) (A∩B)' =A'UB'

That is complement of intersection of two sets is equal to the union of their complements.

Practical Problems

1) If A = {1, 2, 3, 4 }, B = {3, 4,5,6 }
C ={5, 6, 7, 8 } D = {7, 8, 9, 10 }

Find (i) AUB (ii) AUC (iii) BUC
(iv) BUD (v) AUBUC (vi)AUBUD (vii)BUCUD

Solution

(i) AUB = {1, 2, 3, 4, 5, 6 }

(ii) AUC = {1, 2, 3, 4, 5, 6, 7,8 }

(iii) BUC = {3, 4,5, 6, 7, 8 }

(iv) BUD = {3, 4, 5, 6, 7, 8, 9, 10}

(v) AUBUC ={1, 2, 3, 4, 5, 6, 7, 8 }

(vi) AUBUD = {1, 2, 3, 4,5, 6, 7, 8, 9, 10 }

(vii) BUCUD ={3, 4,5, 6, 7, 8, 9, 10 }

2) If A ={1, 3, 5, 7 }, B = { 5, 9, 13, 17 } C = {1, 3, 9, 13 }

Find (i) A∩B (ii) B∩A (iii) A-B (iv) B-A (v) A-C

(vi)(A-B) -C (vii) A-(A-B)

Solution

(i)A∩B = {5}

(ii)B∩A = { 5 }

(iii)A-B ={1,3, 7 }

(iv) B-A = {9, 13, 17}

(v) A-C ={5, 7}

(vi) (A-B) -C) = {7 }

(vii)A-(A-B) = {5 }

3) A = {x: x is a natural number satisfy $1 < x \leq 6$} B = {x: x is a natural number satisfy $6 < x \leq 10$}

Find (i) AUB (ii)A∩B

Solution

A = { 2, 3, 4, 5,6 }

B = {7, 8, 9, 10 }

(i) AUB = {2, 3, 4, 5, 6, 7,8, 9, 10 }

(ii) A∩B = { }

4) Let U = {1,2, 3, 4, 5, 6, 7, 8, 9,10 } and A = {1, 3, 5, 7, 9 } Find A^C.

Solution

A^Cmeans belongs to universe but not in A

A^C = {2, 4, 6, 8, 10}

5) Let U= { 1,2, 3, 4, 5, 6, 7, 8, 9,10 }

A = { 1, 4, 7,10 } B = { 2, 4, 5,8 }

Find A'∩B

Solution

A' = Belongs to universe but not in A

A' = { 2, 3, 5, 6, 8,9 }

A'∩B = { 2, 5,8 }

7) Let A = { 1,2,3 } B = { 2, 4, 5}

C = { 2, 4, 6}, U= { 1, 2, 3, 4, 5, 6, 7,8 }

Verify that (i) (A∩B) = A'∩B'

(ii) (A∩B) = A'UB'

Solution

(i) $(AUB)' = U - (AUB)$

$AUB = \{ 1, 2, 3, 4, 5\}$

$(AUB)' = \{ 6, 7, 8\}$

$A' = U - A = \{ 4, 5, 6, 7, 8\}$

$B' = U - B = \{ 1, 3, 6, 7, 8\}$

$A' \cap B' = \{ 6, 7, 8\}$

Hence $(AUB)' = \underline{\underline{A' \cap B'}}$

(ii) $(A \cap B)' = U - (A \cap B)$

$(A \cap B) = \{ 2\}$

$(A \cap B)' = \{ 1, 3, 5, 6, 7, 8\}$

$A'UB' = \{ 1, 3, 5, 6, 7, 8\}$

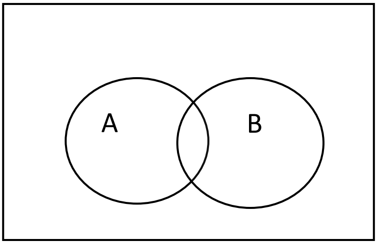
Hence $(A \cap B)' = A'UB'$

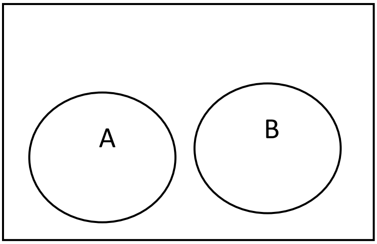
Venn Diagram

The relationship between sets can be represented by means of diagrams. It is known as Venn diagram. It consists of a rectangle and circles. Rectangle represents the universal set and circle represents any set.

For example AUB, A∩B, A-B, and A^{C}can be represented as follows:

1. AUB 1 2





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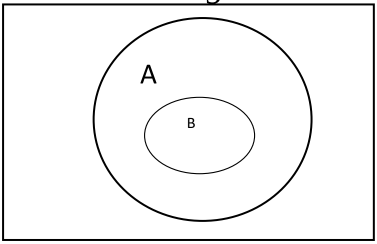
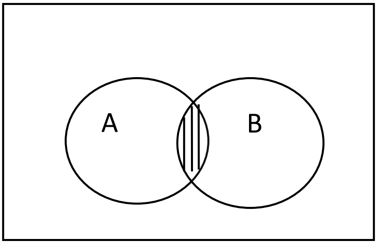


In the first diagram A and B are intersecting in the second diagram, A and B are disjoint and in the third figure, B is a subset of A. In all the diagrams, AUB is equal to the shaded area.

2. A∩B

1



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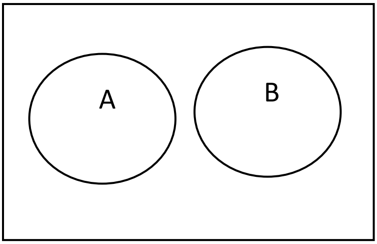

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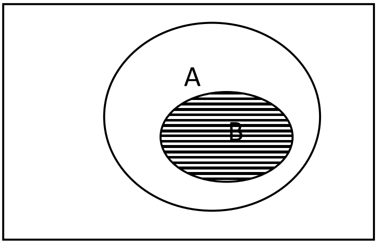
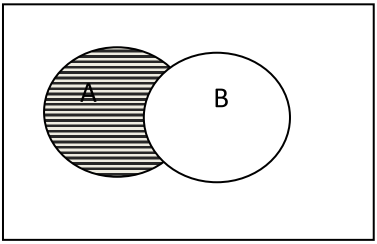


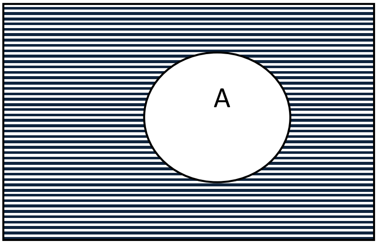
In first diagram A∩B is marked by lines. In the second diagram A and B are disjoint and therefore there is no intersection and so A∩B = Ø. In the third diagram B is a subset of A and A∩B isalso marked by lines.

3. A-B



A – B ie belongs to A but not in B isshaded by lines

4. A^c



A^c i.e. belongs to universe butnot in A is shaded by lines

Theorems on Number of Elements in a Set.

(i)n(AUB) = n(A) + n(B) – n(A∩B)

(ii) n(AUBUC) = n(A) + n(B) + n(C) – n(A∩B) – n(A∩C) – n(B∩C) + n(A∩B∩C)

AUB = at least one of them

A∩B = both A & B

AUBUC = At least one of them

A∩B∩C = All of them

1. Among 60 people, 35 can speak in English, 40 in Malayalam and 20 can speak in both the languages. Find the number of people who can speak at least one of the languages. How many cannot speak in any of these languages?

Solution

n(A) = Speak in English

n(B) = Speak in Malayalam

Given

n(A) = 35, n(B) = 40

$n(A \cap B) = 20$

AUB = (ie people who speak in at least one of the language)

$= n(A) + n(B) – n(A \cap B)$

$= 35 + 40 - 20 = 55$

Number of people who cannot speak in any one of these language = 60- 55 = 5

2. Each student in a class, studies at least one of the subject English, Mathematics and Accountancy. 16 study English, 22 Accountancy and 26 Mathematics. 5 study English and Accountancy, 14 study Mathematics and Accountancy and 2 English, Accountancy and Mathematics. Find the number of student who study

i. English & Mathematics

ii. English, Mathematics but not Accountancy

Solution

Let A = students study English

B = students study Mathematics

C = students study Accountancy

Given

$n(A\ ) = 16,\ n(B)\ 26,\ n(C)\ 22$

$n(A\cap C) = 5,\quad n\ (B\cap C) = 14,\qquad n(A\cap B)?$

$n(A\cap B\cap C) = 2,\qquad n(AUBUC) = 40$

We know that

$n(AUBUC) = n(A) + n(B) + n(C) – n(A\cap B) –n(A\cap C) –n(B\cap C) + n(A\cap B\cap C)$

$40 = 16 + 26 + 22 -\ n(A\cap B) – 5 – 14 + 12$

$n(A\cap B) = 16 + 26 + 22 – 5 – 14 + 2 – 40$

$= 7$

∴Number of students study for English & Mathematics = 7

Number of student who study English, Mathematics but not Accountancy= $n(A\cap B\cap C')$ $n(A\cap B\cap C') = n(A\cap B)-n(A\cap B\cap C)$

$= 7 – 2 = 5$

Number of student who study English, Mathematics and not Accountancy = 5

3. In a college there are 20 teachers, who teach Accountancy or Statistics. Of these 12, teach Accountancy and 4 teach both Statistics and Accountancy. How many teach Statistics?

Solution

Let n(A) = teachers teach Accountancy

n(B) = teacher teach Statistics

Given

n(A) = 12, n(B) ?

$n(A\cap B) = 4$, n(AUB) =20

n(AUB) = $n(A) + n(B) - n(A\cap B)$

20 = 12 + n(B) – 4

n(B) = 20 – 12 + 4 = 12

Number of teachers teach Statistics = 12

4. Out of 2400 students who appeared for BCom degree Examination, 1500 failed in Numerical skills, 1200 failed in Accountancy and 1200 failed in Informatics, 900 failed in both Numerical skills and Accountancy 800 failed in both Numerical skills and Informatics, 300 failed in Accountancy and Informatics, 40 failed in all subjects. How many students passed allthree subjects?

Solution

Let A = number of students failed in Numerical Skills

B = number of students failed in Accountancy

C = number of students failed in Informatics

Given

n(A) = 1500, n(B) = 1200, n(C) = 1200

$n(A\cap B) = 900$, $n(A\cap C)= 800$, $n(B\cap C) = 300$

$n(A \cap B \cap C) = 40$

Number of st udents failed in at least one subject

$= n(AUBUC)$ $n(AUBUC) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 1500 + 1200 + 1200 – 900 – 800 – 300 + 40 = 1940$

Number of student passed in all subjects = 2400 – 1940 = 460

Module V: Theoretical Distribution

Probability distribution (Theoretical Distribution) can be defined as a distribution obtained for a random variable on the basis of a mathematical model. It is obtained not on the basis of actual observation or experiments, but on the basis of probability law.

Random variable

Random variable is a variable who value is determined by the outcome of a random experiment. Random variable is also called chance variable or stochastic variable.

For example, suppose we toss a coin. Obtaining of head in this random experiment is a random variable. Here the random variable of "obtaining heads" can take the numerical values.

Now, we can prepare a table showing the values of the random variable and corresponding probabilities. This is called probability distributions or theoretical distribution. In the above, example probability distribution is :-

Obtaining of heads X	Probability of obtaining heads P(X)
0	$\frac{1}{2}$
1	$\frac{1}{2}$

	$\sum P(X) = 1$

Properties of Probability Distributions:

Every value of probability of random variable will be greater than or equal to zero. i.e., P(X) 0

i.e., P(X) Negative value

Sum of all the probability values will be 1

$\sum P(X) = 1$

Question

A distribution is given below. State whether this distribution is a probability distribution.

X:	0	1	2	3	4
P(X):	0.01	0.10	0.50	0.30	0.90

Solution

Here all values of P(X) are more that zero; and sum of all P(X) value is equal to 1

Since two conditions, namely P(X) 0 and $\sum P(X) = 1$, are satisfied, the given distribution is a probability distribution.

Mathematical Expectation

(Expected Value)

If X is a random variable assuming values x_1, x_2 , x_3 ,…………, x_n with corresponding probabilities P_1 , P_2 , P_3 ,…………, P_n, then the operation of X is defined as $X_1P_1 + X_2 P_2 + X_3 P_3 \ldots\ldots\ldots X_n P_n$

$E(X) = \sum[X.P(X)]$

Question

A petrol pump proprietor sells on an average Rs. 80,000/- worth of petrol on rainy days and an average of Rs. 95.000 on clear days. Statistics from the meteorological department show that the probability is 0.76 for clear weather and 0.24 for rainy weather on coming Wednesday. Find the expected value of petrol sale on coming Wednesday.

Expected Value $E(X) = \sum [X.P(X)]$

$= (80000\times 0.24) + (95000\times 0.76)$

$= 19200+72.200$

= Rs.91400

Question

There are three alternative proposals before a business man to start a new project:-

Proposal I: Profit of Rs. 5 lakhs with a probability of 0.6 or a loss of Rs. 80,000 with a probability of 0.4.

Proposal II: Profit of Rs. 10 laksh with a probability of 0.4 or a loss of Rs. 2 lakhs with a probability of 0.6

Proposal III: Profit of Rs. 4.5 lakhs with a probability of 0.8 or a loss of Rs. 50,000 with a probability of 0.2

If he wants to maximize profit and minimize the loss, which proposal he should prefer?

Solution

Here, we should calculate the mathematical expectation of each proposal.

Expected Value E(X) = ∑ [X.P(X)]

Expected value of Proposal I

= (5,00.000 × 0.6) +(80.000 × 0.4)

= 300,000 – 32,000

= $\underline{\underline{\text{Rs.2, 68,000}}}$

Expected value of Proposal II

= (10,00.000 × 0.4) +(-2,00.000 × 0.6)

= 4,00.000 – 1,20.000

= $\underline{\underline{\text{2,80,000.}}}$

Expected value of Proposal III

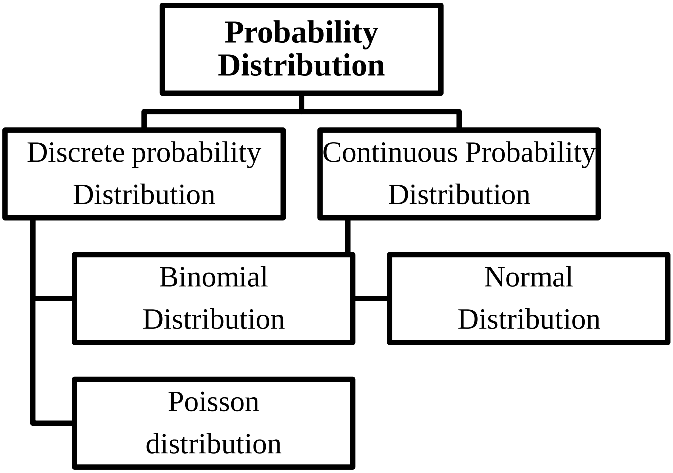
= (4,50,000 × 0.8) +(-50,000 × 0.2)

= 3,60,000 – 10,000

= $\underline{\underline{\text{3,50,000}}}$

Since expected value is highest in case of proposal III, the businessman should prefer the proposal III

Classification of Probability Distribution



Discrete Probability Distribution

If the random variable of a probability distribution assumes specific values only, it is called discrete probability distributions. Binomial distribution and poisson distribution are discrete probability distributions.

Continuous Probability Distributions

If the random variable of a probability distribution assumes any value in a given interval, then it is called continuous probability distributions. Normal distributions is a continuous probability distribution.

Bionomial Distribution

Meaning & Definition:

Binomial Distribution is associated with James Bernoulli, a Swiss Mathematician. Therefore, it is also called Bernoulli distribution. Binomial distribution is the probability distribution expressing the probability of one set of dichotomous alternatives, i.e., success or failure. In other words, it is used to determine the probability of success in experiments on which there are only two mutually exclusive outcomes. Binomial distribution is discrete probability distribution.

Binomial Distribution can be defined as follows: "A random variable r is said to follow Binomial Distribution with parameters n and p if its probability function is:

$$P(r) = {}^{n}C_{r}p^{r}q^{n-r}$$

Where, P = probability of success in a single trial

$q = 1 – p$

n = number of trials

r = number of success in 'n' trials.

Assumption of Binomial Didstribution OR

(Situations where Binomial Distribution can be applied)

Binomial distribution can be applied when:-

1. The random experiment has two outcomes i.e., success and failure.
2. The probability of success in a single trial remains constant from trial to trial of the experiment.
3. The experiment is repeated for finite number of times.
4. The trials are independent.

Properties (features) of Binomial Distribution

1. It is a discrete probability distribution.
2. The shape and location of Binomial distribution changes as ‘p’ changes for a given ‘n’.
3. The mode of the Binomial distribution is equal to the value of ‘r’ which has the largest probability.
4. Mean of the Binomial distribution increases as ‘n’ increases with ‘p’ remaining constant.
5. The mean of Binomial distribution is np.
6. The Standard deviation of Binomial distribution is $\sqrt{npq}$
7. If ‘n’ is large and if neither ‘p’ nor ‘q’ is too close zero, Binomial distribution may be approximated to Normal Distribution.
8. If two independent random variables follow Binomial distribution, their sum also follows Binomial distribution.

Qn: Six coins are tossed simultaneously. What is the probability of obtaining 4 heads?

Sol: P(r) = nC r prqn-r

r = 4

n = 6

p = ½

q = 1 – p = 1 – ½ = ½

∴ p(r = 4) = 6C4 (½)4 (½ $)^{6-4}$

$=\frac{6!}{(6-4)!4!}$! x (½$)^{4+2}$

$= \frac{6!}{2!4!}$! x (½$)^{6}$

$= \frac{6\times5}{2\times1}$ x $\frac{1}{64}$

$= \frac{30}{128}$

= $\underline{\underline{0.234}}$

Qn: The probability that Sachin Tendulkar scores a century in a cricket match is$\frac{1}{3}$. What is the probability that out of 5 matches, he may score century in: -

(1) Exactly 2 matches

(2) No match

Sol: Here p = $\frac{1}{3}$

$q = 1 - \frac{1}{3} = \frac{2}{3}$

P(r) = nC r prqn-r

Probability that Sachin scores centuary in exactly 2 matches is:

$p\ (r = 2) = 5\ C2\ (\frac{1}{3})^2\ (\frac{2}{3})^2$

$= \frac{5!}{(5-2)!2!}\ x\frac{1}{9} \times \frac{8}{27}$

$= \frac{5\times4}{2\times1} \times \frac{1}{279} \times \frac{8}{27}$

$= \frac{160}{486}$

$= \frac{80}{243}$

$= \underline{\underline{0.329}}$

Probability that Sachin scores centuary in no matches is:

$p\ (r = 0) \qquad = 5\ C_0\ (\frac{1}{3})^0\ (\frac{2}{3})^{5-0}$

$= \frac{5!}{(5-0)!0!}\ x1 \times (\frac{2}{3})^5$

$= \frac{5!}{2!\times0!} \times 1\,\frac{32}{243}$

$= \frac{32}{243}$

$= \underline{\underline{0.132}}$

Mean and Standard Deviation of Binomial Distribution

Mean of Binomial Distribution = np

Standard Deviation of Binomial Distribution =$\sqrt{npq}$

Qn: For a Binomial Distribution, mean = 4 and variance = $\frac{12}{9}$. Find n.

Sol: Mean = np = 4

Standard Deviation = $\sqrt{npq}$

∴ Variance = Standard Deviation ^{2}

$=(\sqrt{n}\, pq)^2$

= npq

npq = $\frac{12}{9}$

Divide npq by np to get the value of q

i.e. $= \frac{npq}{np} = q$

q = $\frac{npq}{np} = \frac{\frac{12}{9}}{\frac{4}{1}}$

$= \frac{12}{9} \times \frac{1}{4}$

$= \frac{3}{9} = \frac{1}{3}$

q $= \frac{1}{3}$

p = 1 – q

$= 1 - \frac{1}{3} = \frac{2}{3}$

np = 4

n× $\frac{2}{3}$ = 4

$n \quad = 4 \div \frac{2}{3}$

$n \quad = 4 \times \frac{2}{3}$

$\quad = \underline{\underline{6}}$

<u>Qn:</u> For a Binomial Distribution, mean is 6 and Standard Deviation is √2 . Find the parameters.

Sol: Mean (np) = 6

Standard Deviation ($\sqrt{npq}$) = √2

$\therefore npq = 2$

$\frac{npq}{np} = \frac{2}{6}$

$q = \frac{1}{3}$

$\therefore p = 1 – q$

$= 1 - \frac{1}{3} = \frac{2}{3}$

$np = 6$

$n \text{ x } \frac{2}{3} = 6$

$\therefore n = \frac{6}{\frac{2}{3}} = \quad 6 \text{ x } \frac{3}{2} = \underline{\underline{9}}$

Value of parameters:

$p = \frac{2}{3} \quad q = \frac{1}{3} \quad n = 9$

<u>Fitting a Binomial Distribution</u>

Steps:

Find the value of n, p and q

Substitute the values of n, p and q in the Binomial Distribution function of $nC_rp^rq^{n-r}$

Put r = 0, 1, 2, ……….. in the function $nC_rp^rq^{n-r}$

Multiply each such terms by total frequency (N) to obtain the expected frequency.

Qn: Eight coins were tossed together for 256 times. Fit a Binomi Distribution of getting heads. Also find mean and standa deviation.

Sol: p (getting head) = p = $\frac{1}{2}$

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

$$n = 8$$

Binomial Distribution function is $p(r) = nC_rp^rq^{n-r}$

Put r= 0, 1, 2, 3 …….. 8, then are get the terms of the Binomial Distribution.

No. of heads i.e. r	P (r)	Expected Frequency P (r) x N N = 256
0	$8\ C_0\ (\frac{1}{2})^0\ (\frac{1}{2})^8 = 1\times 1 \times \frac{1}{256} = \frac{1}{256}$	1

1	$8\,C_1\,(\frac{1}{2})^1\,(\frac{1}{2})^7 = 8 \times \frac{1}{256} = \frac{8}{256}$	8
2	$8\,C_2\,(\frac{1}{2})^2\,(\frac{1}{2})^6 = 28 \times \frac{1}{256} = \frac{28}{256}$	28
3	$8\,C_3\,(\frac{1}{2})^3\,(\frac{1}{2})^5 = 56 \times \frac{1}{256} = \frac{56}{256}$	56
4	$8\,C_4\,(\frac{1}{2})^4\,(\frac{1}{2})^4 = 70 \times \frac{1}{256} = \frac{70}{256}$	70
5	$8\,C_5\,(\frac{1}{2})^5\,(\frac{1}{2})^3 = 56 \times \frac{1}{256} = \frac{56}{256}$	56
6	$8\,C_6\,(\frac{1}{2})^6\,(\frac{1}{2})^2 = 28 \times \frac{1}{256} = \frac{28}{256}$	28
7	$8\,C_7\,(\frac{1}{2})^7\,(\frac{1}{2})^1 = 8 \times \frac{1}{256} = \frac{8}{256}$	8
8	$8\,C_8\,(\frac{1}{2})^8\,(\frac{1}{2})^0 = 1 \times \frac{1}{256} = \frac{1}{256}$	1

Mean = np

$= 8\times \frac{1}{2} = 4$

Standard Deviation = $(\sqrt{npq})$

$=\sqrt{8} \times \frac{1}{2} \times \frac{1}{2}$

$= \sqrt{2} = 1.4142$

Poisson Distribution

Meaning and Definition:

Poisson Distribution is a limiting form of Binomial Distribution. In Binomial Distribution, the total numbers of trials are known previously. But in certain real life situations, it may be impossible to count the total number of times a particular event occurs or does not occur. In such cases Poisson Distribution is more suitable.

Poison Distribution is a discrete probability distribution. It was originated by Simeon Denis Poisson.

The Poisson Distribution is defined as:-

$$p\ (r)\ = \frac{e^{-m}\, m^{r}}{r!}\ .$$

Where r = random variable (i.e., number of success in '**n**' trials.

e = 2.7183

m = mean of poisson distribution

Properties of Poisson Distribution

1. Poisson Distribution is a discrete probability distribution.
2. Poisson Distribution has a single parameter 'm'. When 'm' is known all the terms can be found out.
3. It is a positively skewed distribution.
4. Mean and Varriance of Poisson Distribution are equal to 'm'.

5. In Poisson Distribution, the number of success is relatively small.
6. The standard deviation of Poisson Distribution is √m.

Practical situations where Poisson Distribution can be used

1. To count the number of telephone calls arising at a telephone switch board in a unit of time.
2. To count the number of customers arising at the super market in a unit of time.
3. To count the number of defects in Statistical Quality Control.
4. To count the number of bacterias per unit.
5. To count the number of defectives in a park of manufactured goods.
6. To count the number of persons dying due to heart attack in a year.
7. To count the number of accidents taking place in a day on a busy road.

Qn: A fruit seller, from his past experience, knows that 3% of apples in each basket will be defectives. What is the probability that exactly 4 apples will be defective in a given basket?

Sol: $p(r) = \frac{e^{-m} m^r}{r!}$

m = 3

.

.

$\therefore$p (r = 4) = $\frac{e^{--3}\,.\,3^4}{4!} = \frac{0.04979\times81}{4\times3\times2\times1}$

$= \frac{0.04979\times81}{24}$

$= \underline{\underline{0.16804}}$

Qn: It is known from the past experience that in a certain plant, there are on an average four industrial accidents per year. Find the probability that in a given year there will be less than four accidents. Assume poisson distribution.

Sol: p (r<4) = p(r = 0 or 1 or 2 or 3)

= p (r = 0) + p (r =1) + p (r = 2) + p (r = 3)

P (r) $= \frac{e^{-m}\,m^r}{r!}$

m = 4

$\therefore$p (r = 0) = $\frac{e^{-4}\,.\,4^0}{0!} = \frac{0.01832\times1}{1} = 0.01832$

p (r =1) = $\frac{e^{-4}\,.\,4^1}{1!} = \frac{0.01832\times4}{1} = 0.07328$

p (r = 2) = $\frac{e^{-4}\,.\,4^2}{2!} = \frac{0.01832\times16}{2\times1} = 0.14656$

p (r = 3) = $\frac{e^{-4}\,.\,4^3}{3!} = \frac{0.01832\times64}{3\times2\times1} = 0.19541$

$\therefore$p (r < 4) = 0.01832 + 0.07328 + 0.14656 + 0.19541

$= \underline{\underline{0.43357}}$

Qn: Out of 500 items selected for inspection, 0.2% are found to be defective. Find how many lots will contain exactly no defective if there are 1000 lots.

Sol: p = 0.2% = 0.002

n= 500

m = np = 500 x 0.002 = 1

$$P(r) = \frac{e^{-m}\, m^{r}}{r!}$$

$$P(r = 0) = \frac{e^{-1}.\ 1^{0}}{0!} = \frac{0.36788\ \times 1}{1} = 0.36788$$

∴ Number of lots containing no defectives if there are 1000 lots

= 0.36788 x 1000

= 367.88

$= \underline{\underline{368}}$

Qn: In a factory manufacturing optical lenses, there is a small chance of $\frac{1}{1500}$ for any one lense to be defective. The lenses are supplied in packets of 10. Use Poisson Distribution to calculate the approximate number of packets containing (1) one defective (2) no defective in a consignment of 20,000 packets. You are given that $e^{-0.02} = 0.9802$.

Sol: n = 10

p = probability of manufacturing defective lense= $\frac{1}{500}$ = 0.002

m = np = 10 x 0.002 = 0.02

$$p\ (r) = \frac{e^{-m}\, m^{r}}{r!}$$

$$p\ (r = 1) = \frac{e^{-0.02}\ \ 0.02^{1}}{1!} = \frac{0.9802 \times 0.02}{1} = 0.019604$$

∴ No. of packets containing one defective lense

=0.019604 x 20,000

= $\underline{\underline{392}}$

$$p\ (r = 0) = \frac{e^{-0.02}\ \ 0.02^{0}}{0!} = \frac{0.9802 \times 1}{0}$$

= $\underline{\underline{0.9802}}$

∴ No. of packets containing no defective lense = 0.9892 x 20,000

= $\underline{\underline{19604}}$

<u>Qn:</u> A Systematic sample of 100 pages was taken from a dictionary and the observed frequency distribution of foreign words per page was found to be as follows:

No. of foreign words per page (x)	: 0	1	2	3	4	5	6
Frequency (f)	: 48	27	12	7	4	1	1

Calculate the expected frequencies using Poisson Distribution.

<u>Sol:</u> $p\ (r) = \frac{e^{-m}\, m^{r}}{r!}$

Here first we have to find out 'm'

Computation of mean (m)		
x	f	fx
0	48	0
1	27	27
2	12	24
3	7	21
4	4	16
5	1	5
6	1	6
	N = 100	Σfx = 99

$\bar{x} = \frac{\Sigma fx}{N} = \frac{99}{100} = 0.99$

$\therefore$ m = 0.99

$\therefore$ Poisson Distribution = $\frac{e^{-0.99}\,.\,(0.99)^{r}}{r!}$

Computation of expected frequencies		
x	p (x)	Expected frequency Nx p(x)

0	$\frac{e^{-0.99}\,.\ (0.99)^{0}}{0!} = 0.3716$	100 x 0.3716 = 37.2
1	$\frac{e^{-0.99}\,.\ (0.99)^{1}}{1!} = 0.3679$	100 x 0.3679 = 36.8
2	$\frac{e^{-0.99}\,.\ (0.99)^{2}}{2!} = 0.1821$	100 x 0.1821 = 18.21
3	$\frac{e^{-0.99}\,.\ (0.99)^{3}}{3!} = 0.0601$	100 x 0.0601 = 6
4	$\frac{e^{-0.99}\,.\ (0.99)^{4}}{4!} = 0.0149$	100 x 0.0149 = 1.5
5	$\frac{e^{-0.99}\,.\ (0.99)^{5}}{5!} = 0.0029$	100 x 0.0029 = 0.3
6	$\frac{e^{-0.99}\,.\ (0.99)^{6}}{6!} = 0.0005$	100 x 0.0005 = 0.1

Hence, the expected frequencies of this Poisson Distribution are:-

No. of foreign words page :	0	1	2	3	4	5
Expected frequencies (Rounded):	37	37	18	6	2	0

Normal Distribution

The normal distribution is a continuous probability distribution. It was first developed by De-Moivre in 1733 as limiting form of binomial distribution. Fundamental importance of normal distribution is that many populations seem to follow approximately a pattern of distribution as described by normal distribution. Numerous phenomena such as the age distribution of any species, height of adult persons, intelligent test scores of students, etc. are considered to be normally distributed.

Definition of Normal Distribution

A continuous random variable 'X' is said to follow Normal Distribution if its probability function is:

$$P\ (X) = \frac{1}{\sqrt{2\pi\sigma}} \times e\ \frac{1}{2}\ (\frac{x-\ \mu}{\sigma})^2$$

π = 3.146

e = 2.71828

μ = mean of the distribution

σ = standard deviation of the distribution

Properties of Normal Distribution (Normal Curve)

1. Normal distribution is a continuous distribution.
2. Normal curve is symmetrical about the mean.
3. Both sides of normal curve coincide exactly.

4. Normal curve is a bell shaped curve.
5. Mean, Median and Mode coincide at the centre of the curve.
6. Quantities are equi-distant from median.

   $Q_3 – Q_2 = Q_2 – Q_1$
7. Normal curve is asymptotic to the base line.
8. Total area under a normal curve is 100%.
9. The ordinate at the mean divide the whole area under a normal curve into two equal parts.

   (50% on either side).
10. The height of normal curve is at its maximum at the mean.
11. The normal curve is unimodel, i.e., it has only one mode.
12. Normal curve is mesokurtic.
13. No portion of normal curve lies below the x-axis.
14. Theoretically, the range of normal curve is $– \alpha$ to $+ \alpha$.

    But practically the range is $\mu - 3\sigma$ to $\mu + 3\sigma$.

    $\mu \pm 1\sigma$ covers 68.27% area

    $\mu \pm 2\sigma$ covers 95.45% area

    $\mu \pm 3\sigma$ covers 98.73% area.

Importance (or uses) of Normal Distribution

The normal distribution is of central importance in statistical analysis because of the following reasons:-

1. The discrete probability distributions such as Binomial Distribution and Poisson Distribution tend to normal distribution as ‘n’ becomes large.
2. Almost all sampling distributions conform to the normal distribution for large values of ‘n’.
3. Many tests of significance are based on the assumption that the parent population from which samples are drawn follows normal distribution.
4. The normal distribution has numerous mathematical properties which make it popular and comparatively easy to manipulate.
5. Normal distribution finds applications in Statistical Quality Control.
6. Many distributions in social and economic data are approximately normal. For example, birth, death, etc. are normally distributed.

Standard normal probability distribution

A random variable that has a normal distribution with mean ‘0’ and standard deviation 1 is said to have a standard normal probability distribution.

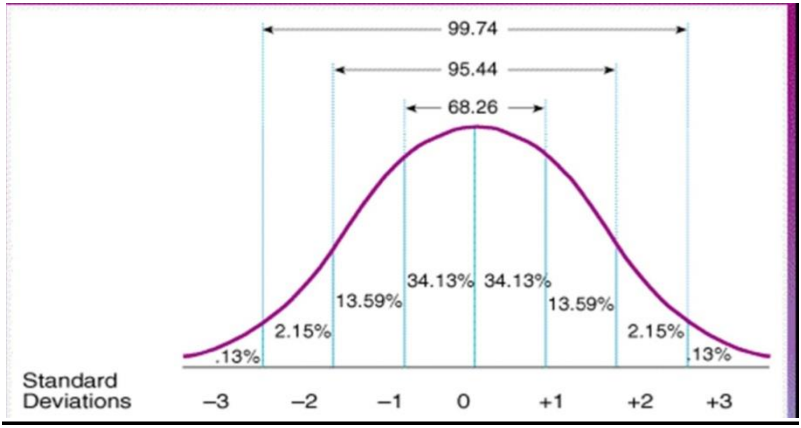
$$Z = \frac{X - \mu}{\sigma}$$

Z score can be defined as the number of standard deviation that a value, x is above or below the mean of the distribution

Using Standard Deviation Units

Because of the equivalence between Z scores and standard deviation units, probabilities of the normal distribution are often expressed as ranges of plus-or-minus standard deviation units.

–1σ to +1σ	0.6826
–2σ to +2σ	0.9545
–3σ to +3σ	0.9973
–6σ to +6σ	0.999999998



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